

George McCandless

THE ABCs OF RBCs

AN INTRODUCTION TO **Dynamic**
Macroeconomic
Models

HARVARD UNIVERSITY PRESS Cambridge, Massachusetts, and London, England
2008

Contents

Preface xiii

Introduction 1

PART ONE BASIC MODELS AND SOLUTION METHODS

- 1 The Basic Solow Model 7**
 - 1.1 The Basic Model 7
 - 1.2 Technological Growth 10
 - 1.3 The Golden Rule 11
 - 1.4 A Stochastic Solow Model 12
 - 1.5 Log-Linear Version of the Solow Model 14
 - 1.5.1 *Capital* 15
 - 1.5.2 *Output* 16
 - 1.6 Reprise 18

- 2 Savings in an OLG Model 19**
 - 2.1 The Basic OLG Model 20
 - 2.1.1 *An Example Economy* 26
 - 2.2 Dynamics 27
 - 2.3 A Stochastic Version 28
 - 2.4 Reprise 32
 - 2.5 Matlab Code Used to Produce Figure 2.2 32

3	Infinitely Lived Agents	33
3.1	A Robinson Crusoe Economy with Fixed Labor	34
3.1.1	<i>Variational Methods</i>	34
3.2	A Robinson Crusoe Economy with Variable Labor	38
3.2.1	<i>The General Model</i>	38
3.2.2	<i>Solution for a Sample Economy</i>	40
3.3	A Competitive Economy	41
3.4	The Second Welfare Theorem	44
3.4.1	<i>An Example Where the Representative Agent Economy and the Decentralized Economy Are Not Equal</i>	45
3.5	Reprise	49
4	Recursive Deterministic Models	50
4.1	States and Controls	51
4.2	The Value Function	52
4.3	A General Version	55
4.4	Returning to Our Example Economy	58
4.4.1	<i>Another Version of the Same Economy</i>	59
4.5	An Approximation of the Value Function	60
4.6	An Example with Variable Labor	63
4.7	Reprise	66
4.8	Matlab Code for Figures 4.2 and 4.3	67
5	Recursive Stochastic Models	69
5.1	Probability	70
5.2	A Simple Stochastic Growth Model	71
5.3	A General Version	74
5.3.1	<i>The Problem of Dimensionality</i>	76
5.4	The Value Function for the Simple Economy	77
5.4.1	<i>Calculating the Value Functions</i>	78
5.5	Markov Chains	80
5.6	Reprise	86
5.7	Matlab Code	87
6	Hansen's RBC Model	89
6.1	Hansen's Basic Model	90
6.2	Log Linearization Techniques	94
6.2.1	<i>The Basics of Log Linearization</i>	95
6.2.2	<i>Uhlig's Method of Log Linearization</i>	98

6.3	Log-Linear Version of Hansen's Model	100
6.3.1	<i>Solution Using Jump Variables</i>	104
6.3.2	<i>Calibration of the Log-Linear Model</i>	106
6.3.3	<i>Variances of the Variables in the Model</i>	109
6.4	Hansen's Model with Indivisible Labor	112
6.4.1	<i>Stationary State</i>	115
6.4.2	<i>Log-Linear Version of the Indivisible Labor Model</i>	118
6.5	Impulse Response Functions	120
6.6	Reprise	124
6.7	Appendix 1: Solving the Log-Linear Model	124
6.8	Appendix 2: Blanchard and Kahn's Solution Method	128
6.8.1	<i>General Version</i>	129
6.8.2	<i>Stochastic Shocks</i>	131
6.8.3	<i>Hansen's Model and Blanchard-Kahn</i>	132
6.8.4	<i>The Generalized Schur Method</i>	134
6.9	Matlab Code	142
6.9.1	<i>Solution to Basic Hansen Model</i>	142
6.9.2	<i>Approximating the Variances</i>	143
6.9.3	<i>Code for Appendix 2</i>	144

7 Linear Quadratic Dynamic Programming 146

7.1	Taylor Approximations of the Objective Function	147
7.2	The Method of Kydland and Prescott	148
7.2.1	<i>An Example</i>	151
7.2.2	<i>Solving the Bellman Equation</i>	154
7.2.3	<i>Calibrating the Example Economy</i>	155
7.3	Adding Stochastic Shocks	157
7.3.1	<i>The Example Economy</i>	160
7.3.2	<i>Calibrating the Example Economy</i>	163
7.4	Hansen with Indivisible Labor	166
7.5	Impulse Response Functions	172
7.5.1	<i>Vector Autoregressions</i>	174
7.6	An Alternative Process for Technology	176
7.7	Reprise	178
7.8	Matlab Code	178

PART TWO EXTENSIONS OF THE BASIC RBC MODEL

8	Money: Cash in Advance	183
8.1	Cooley and Hansen's Model	184
8.2	Finding the Stationary State	190

8.3	Solving the Model Using Linear Quadratic Methods	195
8.3.1	<i>Finding a Quadratic Objective Function</i>	196
8.3.2	<i>Finding the Economy Wide Variables</i>	199
8.4	Solving the Model Using Log Linearization	202
8.4.1	<i>The Log Linearization</i>	202
8.4.2	<i>Solving the Log-Linear System</i>	205
8.4.3	<i>Impulse Response Functions</i>	209
8.5	Seigniorage	210
8.5.1	<i>The Model</i>	212
8.5.2	<i>The Stationary State</i>	215
8.5.3	<i>Log-Linear Version of the Model</i>	218
8.6	Reprise	222
8.7	Appendix 1: CES Utility Functions	223
8.8	Appendix 2: Matrix Quadratic Equations	230
8.9	Matlab Code for Solving the CES Model with Seigniorage	233
9	Money in the Utility Function	236
9.1	The Model	237
9.2	Stationary States	240
9.3	Log-Linear Version of the Model	242
9.4	Seigniorage	246
9.4.1	<i>The Full Model</i>	248
9.4.2	<i>Stationary States</i>	248
9.4.3	<i>Log Linearization</i>	251
9.5	Reprise	256
10	Staggered Pricing Model	258
10.1	The Basic Model	259
10.1.1	<i>The Final Goods Firms</i>	259
10.1.2	<i>The Intermediate Goods Firms</i>	261
10.1.3	<i>The Family</i>	266
10.1.4	<i>Equilibrium Conditions</i>	267
10.1.5	<i>The Full Model</i>	269
10.2	The Stationary State	270
10.3	Log Linearization	273
10.3.1	<i>Log Linearization of the Firm's Problem</i>	273
10.3.2	<i>The Final Goods Pricing Rule</i>	273
10.3.3	<i>The Intermediate Goods Pricing Rule</i>	273
10.3.4	<i>Inflation Equation (Phillips Curve)</i>	275
10.3.5	<i>Log Linear Version of the Model</i>	277
10.4	Solving the Log Linear Model	279
10.4.1	<i>Impulse Response Functions</i>	285

10.5	Inflation Adjustment for Nonoptimizing Firms	290
10.5.1	<i>The Stationary State</i>	291
10.5.2	<i>Log Linearization</i>	293
10.5.3	<i>Solving the Model</i>	295
10.5.4	<i>Impulse Response Functions</i>	300
10.6	Reprise	304
11	Staggered Wage Setting	306
11.1	The Labor Bundler	307
11.1.1	<i>First-Order Conditions for Families</i>	310
11.1.2	<i>The Rest of the Model</i>	312
11.1.3	<i>Equilibrium Conditions</i>	313
11.1.4	<i>The Full Model</i>	314
11.2	The Stationary State	315
11.3	Log Linearization	317
11.4	Solving the Model	321
11.4.1	<i>Impulse Response Functions</i>	325
11.5	Reprise	327
12	Financial Markets and Monetary Policy	329
12.1	Working Capital	331
12.1.1	<i>Households</i>	331
12.1.2	<i>Firms</i>	333
12.1.3	<i>Financial Intermediaries</i>	334
12.1.4	<i>The Full Model</i>	334
12.1.5	<i>The Stationary State</i>	336
12.1.6	<i>Log Linear Version of the Model</i>	340
12.1.7	<i>Impulse Response Functions</i>	345
12.1.8	<i>Economy with Annual Inflation of 100 Percent</i>	347
12.1.9	<i>Comparative Impulse Response Functions</i>	349
12.2	Central Banking and Monetary Policy Rules	352
12.2.1	<i>The Model with a Taylor Rule</i>	353
12.2.2	<i>Stationary States</i>	356
12.2.3	<i>Log-Linear Version and Its Solution</i>	357
12.2.4	<i>Comparing a Taylor Rule to a Friedman Rule</i>	362
12.3	Reprise	368
13	Small Open Economy Models	370
13.1	The Preliminary Model	371
13.1.1	<i>The Household</i>	371
13.1.2	<i>The Firm</i>	374
13.1.3	<i>Equilibrium Conditions</i>	374

13.1.4	<i>Stationary State</i>	374
13.1.5	<i>The Dynamic (Log-Linear) Model</i>	376
13.2	Model with Capital Adjustment Costs	379
13.3	Closing the Open Economy	386
13.3.1	<i>Interest Rates and Country Risk</i>	386
13.3.2	<i>The Dynamic Version</i>	388
13.4	The “Closed” Open Economy with Money	393
13.4.1	<i>The Open Economy Conditions</i>	394
13.4.2	<i>The Household</i>	395
13.4.3	<i>Firms</i>	396
13.4.4	<i>Equilibrium Conditions</i>	396
13.4.5	<i>The Full Model</i>	397
13.4.6	<i>The Stationary State</i>	398
13.4.7	<i>Log-Linear Version of Full Model</i>	400
13.5	Reprise	410
References		411
Index		417