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# Voting Procedures for Electing a Single Candidate

Proving Their (In)Vulnerability to Various  
Voting Paradoxes

 Springer

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