An Introduction to Modelling Buckling and Collapse
Second Edition

Authors:
Brian G. Falzon
Dennis Hitchings

ISBN 1 874 376 18 2

© 2006, 2007 NAFEMS

ACKNOWLEDGEMENT

I wish to take this opportunity, as Chairman of NAFEMS Computational Structural Mechanics Working Group, to thank the Working Group members for their help and support in the presentation of this document and, in particular, Alan Prior, who acted as Chief Reviewer.

The composition of the Computational Structural Mechanics Working Group is:

Peter Bartholomew
Adib Becker
Ed DeVries
Brian Falzon
Peter Gosling
Peter Hopkins
Geir Horrignhe
David Irving
Steve King
Nigel Knowles
John McMee
Navin Prinja
Alex Ramsey
Nick Warriner
Keith Wright
Louise Wright

Peter Bartholomew
Independent Consultant
Adib Becker
University of Nottingham
Ed DeVries
MSC.Software
Brian Falzon
Imperial College
Peter Gosling
University of Newcastle
Peter Hopkins
Astrium
Geir Horrignhe
Not
David Irving
FEA Ltd
Steve King
Abaqus UK
Nigel Knowles
Independent Consultant Chairman
John McMee
QinetiQ Co-Chairman
Navin Prinja
NNC-Amec
Alex Ramsey
Independent Consultant
Nick Warriner
University of Nottingham
Keith Wright
Structural Integrity Assessments
Louise Wright
NPL

Nigel Knowles
Chairman
Contents

1. Introduction .............................................................................................................. 1
   1.1. Scope .................................................................................................................. 2
   1.2. Readership ......................................................................................................... 2
   1.3. Layout ................................................................................................................ 2

2. A Brief overview of elastic stability .................................................................... 5
   2.1. Introduction ....................................................................................................... 5
   2.2. Stiffening behaviour ......................................................................................... 5
   2.3. ‘Snap-through’ behaviour ................................................................................ 5
   2.4. Classical buckling ............................................................................................. 8
   2.5. Stable symmetric buckling .............................................................................. 12
   2.6. Unstable symmetric buckling .......................................................................... 14
   2.7. Asymmetric buckling ....................................................................................... 16

3. Linear buckling analysis ..................................................................................... 19
   3.1. Introduction ....................................................................................................... 19
   3.2. Linear buckling analysis .................................................................................. 19
   3.3. Steps for a finite element eigenvalue analysis .................................................. 22
   3.4. Methods for finding eigenvalues and eigenvectors .......................................... 23
   3.5. Example 3A – linearised plate buckling ......................................................... 24
   3.6. Buckling analysis for pre-loaded structures ...................................................... 27
   3.7. Symmetry .......................................................................................................... 28
   3.8. Example 3B – Using symmetry for isotropic plate buckling ......................... 28

4. Geometric non-linear analysis ........................................................................... 32
   4.1. Introduction ....................................................................................................... 35
   4.2. Non-linear analysis formulation ...................................................................... 36
   4.3. Newton-Raphson methods ............................................................................. 37
   4.4. Limitation of Newton-Raphson schemes ....................................................... 41
   4.5. Direct load incrementation scheme .................................................................. 42
   4.6. Energy dissipation scheme .............................................................................. 43
   4.7. Arc-length methods ......................................................................................... 44
   4.8. Follower force loading .................................................................................... 47
   4.9. Example 4A – Clamped curved beam with central point load ....................... 47
   4.10. Example 4B – Postbuckling stiffened carbon-fibre composite panels .......... 51

5. Dynamic analysis for solving non-linear problems ........................................... 59
   5.1. Introduction ....................................................................................................... 59
   5.2. Dynamic solution methods for non-linear problems ........................................ 59
   5.3. Implicit solution ................................................................................................ 61
   5.4. Explicit solution ................................................................................................ 62
   5.5. Other forms of explicit/implicit solution methods ............................................ 63
   5.6. Choice of solution method ............................................................................... 64
   5.7. Mass-scaling ..................................................................................................... 64
   5.8. Load rate ........................................................................................................... 65
   5.9. Extended explicit solution method ................................................................. 65
   5.10. Combined quasi-static/pseudo-transient method ............................................ 67
5.11. Example 5A - Clamped curved beam with central point load using explicit dynamic analysis
5.12. Example 5B - Postbuckling response of a cantilever beam using explicit dynamic analysis
5.13. Example 5C - Clamped curved beam with central point load using a combined quasi-static/pseudo-transient method
5.14. Example 5D - Postbuckling of a blade-stiffened panel using explicit analysis
5.15. Example 5E - Postbuckling of an I-stiffened panel using combined quasi-static/pseudo-transient method
5.16. Remarks

6. Modelling collapse through material plasticity
6.1. Introduction
6.2. Basic plasticity
6.3. Limit load analysis (plastic collapse)
6.4. Application to structural collapse - limit load analysis
6.5. General elastic-plastic analysis
6.6. Finite element modelling for a plastic collapse analysis
6.7. Mesh and element considerations
6.8. Other considerations
6.9. Example 6A - Finite element plastic collapse solution

7. Collapse due to fracture
7.1. Introduction
7.2. Fracture mechanics basics
7.3. Example 7A - Stress intensity factor calculation
7.4. Quarter-point isoparametric elements
7.5. Virtual crack closure technique
7.6. Interface elements
7.7. Example 7B - VCCT applied to skin-stiffener debonding problem
7.8. Example 7C - Mixed-mode multiple delamination using interface elements

8. Practical guide to selecting a solution scheme

9. Concluding remarks

10. References

11. Appendix: Solution procedures for eigenproblems
11.1. Subspace iteration
11.2. Lanczos method

12. Glossary