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Flow and Transport in Porous Media and Fractured Rock
From Classical Methods to Modern Approaches
Second, Revised and Enlarged Edition
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