

Quantum Statistical Field Theory

An Introduction to Schwinger's Variational Method
with Green's Function Nanoapplications, Graphene
and Superconductivity

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