

An Introduction to Graphene Plasmonics

P.A.D Gonçalves

University of Minho, Portugal

N.M.R Peres

Univeristy of Minho, Portugal

 **World Scientific**

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI • TOKYO

Contents

<i>Foreword</i>	vii
<i>Preface</i>	ix
<i>Acknowledgments</i>	xiii
<i>List of Figures</i>	xvii
<i>List of Tables</i>	xix
1. Introduction	1
1.1 Plasmonics: Generalities	1
1.2 Plasmonics: Recent Developments	3
2. Electromagnetic Properties of Solids in a Nutshell	11
2.1 Classical Electrodynamics Basics	11
2.1.1 Maxwell's equations	11
2.1.2 Boundary conditions	14
2.2 Drude Model	14
2.3 Preliminaries to Graphene Plasmonics	17
2.3.1 Elementary electronic properties	17
2.3.2 The optical conductivity of graphene	24
2.3.3 Lindhard function: Beyond the local approximation	28
2.3.4 Lindhard polarization function in relaxation-time approximation: The case of graphene	33
2.4 The Transfer-Matrix Method and the First Appearance of Plasmons in Graphene	35
2.4.1 Transfer-matrix for a graphene monolayer	36

2.4.2	Transmittance, reflectance and absorbance of electromagnetic radiation by a graphene monolayer . . .	38
2.4.3	Transfer-matrix for a graphene double-layer	40
2.4.4	Transfer-matrix for multi-layer graphene structures	42
2.4.5	Plasmons: A transfer-matrix approach	45
3.	Surface Plasmon-Polaritons at Dielectric-Metal Interfaces	47
3.1	Single Dielectric-Metal Interface	47
3.1.1	Dispersion relation	48
3.1.2	Propagation length and field confinement	54
3.2	Multilayer Structures	57
3.2.1	Double Interface	57
3.2.2	Dispersion relation	59
3.3	A Short Note on Perfect Conductors	64
4.	Graphene Surface Plasmons	67
4.1	Monolayer Graphene	67
4.1.1	Spectrum of GSPs	68
4.1.2	Field profile of GSPs in single-layer graphene	75
4.1.3	Plasmon dispersion revisited	78
4.1.4	Loss function	81
4.1.5	A word on TE surface waves	85
4.2	Double-Layer Graphene	86
4.2.1	Dispersion relation	87
4.2.2	Field profile of GSPs in double-layer graphene	91
4.2.3	Plasmon dispersion beyond the Drude model approximation	94
4.3	Surface Plasmon-Phonon-Polaritons in Graphene	101
4.3.1	Graphene on SiC	103
4.3.2	Graphene on SiO ₂	106
4.4	Magneto-Plasmons in Monolayer Graphene	112
4.4.1	Derivation of the spectrum's condition	113
4.4.2	Solution of the semi-classical spectrum's condition in a special limit	115
4.4.3	Spectrum of magneto-plasmons in the quantum regime: Landau quantization	120
4.5	A Detour: Surface Phonon-Polaritons in hBN	124
4.5.1	Solution of the electromagnetic problem	125

4.5.2	Guided phonon-polariton modes	128
5.	Excitation of Graphene Surface Plasmons	131
5.1	Grating Coupling	131
5.1.1	Graphene on Gallium-Arsenide: The role of SO phonons	132
5.2	Prism Coupling	134
5.2.1	Otto configuration: Single-layer graphene	134
5.3	Near-field Excitation and Imaging of GSPs	140
5.4	Others	142
5.5	Excitation of SPP's by a Moving Line of Charge	143
6.	Launching Plasmons Using a Metallic Antenna	149
6.1	Theoretical Model and Integral Equation	149
6.2	Approximate Solution via Fourier Expansion	153
6.3	Results and Discussion	156
7.	Plasmonics in Periodic Arrays of Graphene Ribbons	163
7.1	A Seminal Paper	163
7.2	Theoretical Model	167
7.2.1	Setting up the model	167
7.2.2	The scattering problem	170
7.3	Applications and Results	172
7.3.1	Periodic array of graphene ribbons	172
7.3.2	Theory <i>versus</i> experiment	178
7.4	A THz Polarizer	183
7.5	Scattering From a Periodic Grid in the Regime $kw < 1$	185
8.	Plasmons in Graphene Nanostructures and in One-dimensional Channels	193
8.1	Edge Plasmons in a Graphene Nanoribbon	195
8.1.1	Prelude: Green's functions for a 2-layered medium	195
8.1.2	Plasmonic spectrum of a graphene nanoribbon	199
8.1.3	An extension: magneto-plasmons in a graphene ribbon	210
8.1.4	Scattering of THz radiation by a graphene micro-ribbon	211
8.2	Localized Surface Plasmons in a Graphene Ring	218

8.2.1	Spectrum of graphene plasmons in a graphene ring	218
8.3	Plasmonic Excitations in a Graphene Nanodisk	232
8.4	Scattering of Graphene Plasmons by a Conductivity Step	239
8.4.1	The mathematical problem	241
8.4.2	Solution of the Riemann-Hilbert problem	242
8.4.3	Further details	244
8.4.4	Calculation of the reflection amplitude	245
8.4.5	An alternative derivation of Eq. (8.137)	247
8.5	Localized Surface Plasmons in a Graphene Sheet with a Gaussian Groove	250
8.5.1	Few useful definitions	251
8.5.2	Formulation of the problem	253
8.5.3	Green's theorem, Green's functions, and the eigen- value problem	254
8.5.4	Spectrum of surface plasmon-polaritons in the presence of a Gaussian groove	259
9.	Excitation of Surface Plasmon-Polaritons Using Dielec- tric Gratings	267
9.1	Some Basic Definitions and Results	268
9.2	Tangent and Normal Vectors, and Boundary Conditions .	270
9.3	A Trivial Example: Recovering Previous Results	271
9.4	The Fields in D_+ and D_-	272
9.4.1	Reflectance and transmittance efficiencies	274
9.4.2	Particular limits for the transmittance and the re- flectance	274
9.5	A Non-Trivial Example: A Grating with a Sine-Profile . .	275
10.	Excitation of Plasmons by an Emitting Dipole	281
10.1	Statement of the Problem and a Bit of Electrostatics . . .	282
10.2	Calculation of the Non-Radiative Transition Rate: Particle-Hole Excitations Pathway	287
10.3	Calculation of the Total Transition Rate: Full Electromag- netic Calculation	289
10.4	Purcell Effect in Hyperbolic Materials	292
11.	Concluding Remarks	295
Appendix A	Derivation of the Susceptibility of Graphene	301

A.1	Undoped Susceptibility	301
A.1.1	The imaginary part of the <i>undoped</i> susceptibility	302
A.1.2	The real part of the <i>undoped</i> susceptibility	303
A.2	Doped Part: Fermi Sea Contributions	303
A.2.1	Intraband term	303
A.2.2	Interband term	304
A.2.3	Imaginary part of χ_D^1	306
A.2.4	Imaginary part of χ_D^2	306
A.2.5	Real Part of χ_D^1 and χ_D^2	307
A.3	Summary for the Real and Imaginary Parts of Graphene Susceptibility	309
A.3.1	Real part of the susceptibility	310
A.3.2	Imaginary part of the susceptibility	311
A.4	Some Useful Integrals	312
Appendix B	Derivation of the Intra- and Inter-band Conductivity of Graphene	313
Appendix C	Inhomogeneous Drude Conductivity	319
C.1	Longitudinal Conductivity	320
C.2	Transverse Conductivity	321
Appendix D	Derivation of the Expression Relating the Longitudinal Conductivity with the Polarizability	323
Appendix E	Derivation of the Polarization of Graphene in the Relaxation-Time Approximation	327
E.1	Hamiltonian and Particle and Current Densities	327
E.2	Local Equilibrium Density Matrix	328
E.3	A Useful Identity	329
E.4	Equation of Motion for the Density Matrix	330
E.5	Polarization in the Relaxation-Time Approximation	331
Appendix F	RPA for Double-Layer Graphene	333
F.1	Equation of Motion Method	333
F.2	Diagrammatic Approach	337
Appendix G	Effective Dielectric Constants for Coulomb-	

Coupled Double-Layer Graphene	339
Appendix H Magneto-Optical Conductivity of Graphene	343
Appendix I Supplementary Material for Chapter 7	347
I.1 Derivation of the Expressions for the Transmittance and Reflectance	347
I.2 Modes Contributing to Each Resonance	349
Appendix J The Method of Toigo	351
Appendix K Boundary Condition in the Dipole Problem	355
Appendix L Fourier Transform of the Dipole Potential	359
Appendix M Non-Radiative Decaying Rate: Electrostatic Calculation	361
M.1 Doped Graphene	361
M.2 A Detour: Neutral Graphene	363
Appendix N Reflection Amplitude of an Electromagnetic Wave due to a Graphene Interface	367
N.1 The Simple Case of Free Standing Graphene	367
N.2 Reflection and Transmission Amplitudes: Fresnel Coefficients	369
N.2.1 TE (transverse electric) / s -components	372
N.2.2 TM (transverse magnetic) / p -components	373
Appendix O Green's Functions in the Rope Problem	375
O.1 The Homogeneous Rope	375
O.1.1 Response to a localized force	375
O.2 The Inhomogeneous Rope	377
O.2.1 Reflection and transmission coefficients	377
O.2.2 Response to an external force in the inhomogeneous regime	378
Appendix P Derivation of the Transition Rate of a Quantum Emitter Near an Interface	383
P.1 Polarization Vectors and Green's Functions	384

P.1.1	Definition of s - and p -polarization vectors and angular spectrum representation	384
P.1.2	The free space Green's function	386
P.1.3	Green's functions for problems with flat interfaces I	388
P.2	Dipole Decaying Rate	391
P.3	Explicit Form of the Tensor Product of the Polarization Vectors	392
P.4	The Free Green's Function in Real Space	393
P.5	Green's Function for Problems With Flat Interfaces II	394
P.5.1	Free-space Green's function revisited	394
P.5.2	Electric field from a dipole in terms of Green's functions	395
P.5.3	Green's functions: s -component	396
P.5.4	Green's functions: p -component	398
P.5.5	Derivation of the matrix structure of the Green's functions	399
 <i>Bibliography</i>		403
 <i>Index</i>		427