SUPERCONDUCTIVITY
A New Approach Based on the Bethe–Salpeter Equation in the Mean-Field Approximation

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Contents

Preface vii

More Frequently Used Abbreviations in the Text xi

Symbols Used in More than One Chapter xiii

Units: cgs and Natural xv

1. The Bethe–Salpeter Equation (BSE) 1
   1.1 Introduction ............................................. 1
   1.2 BSE for Two Spin-zero Particles Interacting via Another
        Spin-zero Particle in the Ladder Approximation .... 2
   1.3 BSE for Scalar Particles in Momentum Space ....... 5
   1.4 Remarks .................................................... 8
   Notes and References ....................................... 8

2. Customization of the Bethe–Salpeter Equation (BSE)
   to Superconductivity 9
   2.1 Introduction ............................................. 9
   2.2 BSE for Two Spin-1/2 Particles Interacting via a
        Spin-zero Particle in the Ladder Approximation^2 .... 9
        2.2.1 IA .................................................. 10
        2.2.2 BSE in IA .......................................... 10
2.3 Temperature-generalization of $T = 0$ BSE via Matsubara Technique .......................... 14
2.4 Temperature-generalization of (2.8 a–d) via (2.9 a–d ) ........................................... 15
2.5 Remarks ........................................... 17
Notes and References ........................................... 18

3. Re-derivation of Some Well-Known Results of BCS Theory via BSE-Based Approach 19
3.1 Introduction ........................................... 19
3.2 Cooper Problem ........................................... 19
3.3 From Cooper Problem to Major Results of BCS Theory at $T = 0$: A Qualitative Account ........................................... 21
3.3.1 The reduced BCS wave function ........................................... 21
3.3.2 The reduced BCS Hamiltonian ........................................... 22
3.3.3 Determination of $u_k$ and $v_k$ and the equation for $T = 0$ gap ........................................... 23
3.3.4 Interpretation of $\Delta_k$ and $E_k$ defined in Eq. (3.9) ........................................... 25
3.3.5 Temperature-generalization of BCS theory ........................................... 26
3.4 Cooper Problem and BCS Theory Revisited via BSE-based Approach ........................................... 28
3.4.1 Cooper problem ........................................... 28
3.4.2 The equation for $T_c$ ........................................... 29
3.4.3 An equation equivalent to the BCS equation for $\Delta(T)$ ........................................... 30
3.4.4 Implications of Eqs. (2.14 b) and (2.14 c) obtained in the previous chapter ........................................... 31
3.4.5 Implications of Eq. (2.14 d) obtained in the previous chapter ........................................... 32
3.5 Remarks ........................................... 32
Notes and References ........................................... 36
4. Generalized BCS Equations for Superconductors Characterized by High-\(T_c\)s and Multiple Gaps  
4.1 Introduction ........................................ 41  
4.2 Superpropagator .................................... 42  
4.3 Multiple Debye Temperatures ....................... 44  
4.4 Generalized BCS Equations (GBCSEs) for Composite SCs ........................................ 45  
4.5 The Bogoliubov Constraint .......................... 47  
4.6 Application of GBCSEs to High-\(T_c\) SCs: General Considerations .............................. 48  
4.7 Application of GBCSEs to Specific High-\(T_c\) SCs ......................... 49  
4.7.1 \(\text{MgB}_2\) ........................................ 49  
4.7.2 \(\text{YBa}_2\text{Cu}_3\text{O}_7\) ................................ 50  
4.7.3 \(\text{Tl}_2\text{Ba}_2\text{CaCu}_2\text{O}_8\) .............................. 52  
4.7.4 \(\text{Tl}_2\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}\) .................................. 52  
4.7.5 \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8\) and \(\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}\) ................. 53  
4.7.6 \(\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2\) ................................ 55  
4.8 Remarks ............................................. 56  
Notes and References .................................... 64  

5. Multi-Gap Superconductivity: Generalized BCS Equations (GBCSEs) as an Alternative to the Approach Due to Suhl, Matthias, and Walker (SMW)  
5.1 Introduction .......................................... 67  
5.2 A Review of the SMW Approach for a Two-gap Superconductor ............................... 67  
5.3 Application of SMW Approach to \(\text{MgB}_2\) ........................................ 71  
5.4 A Quantitative Study of \(\text{MgB}_2\) via GBCSEs ........................................ 73  
5.5 Remarks ............................................. 74  
Notes and References .................................... 77
6. **Thermal Conductivity of MgB$_2$**

6.1 Introduction ............................................. 79
6.2 Features of Superconducting State that Affect Thermal Conductivity ............................................. 80
6.3 Equations for $\kappa_{es}$ and $\kappa_{gs}$ in the G and GK Theory ............................................. 81
6.4 On the Input of $\Delta(T)$ of MgB$_2$ into the Equations for $\kappa_{es}$ and $\kappa_{gs}$ in the G and GK theory ............................................. 82
6.5 Addressing the Thermal Conductivity Data on MgB$_2$ Obtained by B et al. via the G$^1$ and GK$^2$ Theory and GBCSEs ............................................. 83
6.6 The Data Obtained by Schneider et al. ............................................. 87
6.7 Remarks ............................................. 87
Notes and References ............................................. 91

7. **Dynamical Equations for Temperature-Dependent Critical Magnetic Fields**

7.1 Introduction ............................................. 93
7.2 Incorporation of an External Magnetic Field into the $T$-dependent Gap Equation for Elemental SCs ............................................. 94
7.3 Superconducting Features of Some Elements in the $T = T_c$, $H = 0$ and $T = 0$, $H = H_0$ States ............................................. 98
7.4 The $T \neq 0$, $H \neq 0$ Equation: The BCS Regime ............................................. 100
7.5 Equations for $T$-dependent Critical Magnetic Fields of a Composite SC ............................................. 102
7.6 The $T \neq 0$, $H \neq 0$ Equation: Beyond the BCS Regime ............................................. 106
7.7 de Haas–van Alphen Oscillations ............................................. 108
7.8 Remarks ............................................. 110
Notes and References ............................................. 111

8. **Dynamics-based Equations for Critical Current Densities**

8.1 Introduction ............................................. 113
8.2 Equations for $P_c(T)$ for an Elemental SC in the Absence of an External Magnetic Field ............................................. 115
8.3 Calculation of $P_0$ and the Number Density of Electrons for Sn From the Value of its $j_0$ 119
8.4 Equations for $P_c(T)$ and $P_0$ for a Non-elemental SC in the Absence of an External Magnetic Field 123
8.5 Remarks 124
Notes and References 125

9. BCS–BEC Crossover Physics Without Appeal to Scattering Length Theory 127

9.1 Introduction 127
9.1.1 The need to incorporate $E_F$ into the equations for $\Delta(T)$ and $T_c$ 127
9.1.2 The meaning of crossover 128
9.1.3 Salient features of crossover via SLT 129
9.1.4 Scope of this chapter 129
9.2 Framework of Crossover: The Usual BCS Equations 131
9.3 Crossover via Eq. (9.8) 133
9.4 Procedural details about calculations 134
9.5 Crossover via Alternative Equations 136
9.6 Physical Significance of $\rho$ and Variation of $\rho$ as $n$ is Decreased 138
9.7 Remarks 140
Notes and References 143

10. On the Puzzle Posed by Superconducting SrTiO$_3$ 145

10.1 Introduction 145
10.1.1 The nature of the puzzle 145
10.1.2 Earliest work on the puzzle 145
10.1.3 Recent findings about SrTiO$_3$ 147
10.2 Dealing with SrTiO$_3$ via Equations Incorporating Chemical Potential 147
10.3 Solutions of Eqs. (10.5) and (10.14) as $\mu$ is Varied 151
10.3.1 The Debye temperature of Ti ions 151
10.3.2 Fixing values of $\mu_1$ for which Eqs. (10.5) and (10.14) need to be solved 152
10.3.3 Equations (10.5), (10.10) and (10.14) comprise an under-determined system; approximate solutions for LHS of the dome (2.5 meV ≤ μ₁ < 15 meV) with suitable assumptions .................................. 153

10.3.4 Approximate solutions for RHS of the dome (15 meV < μ ≤ 45 meV) .......................... 155

10.4 A Purely Mathematical Model to Address the Tᵋ versus n plot in Ref. 2 .......................... 156

10.5 On the Feature of Two Domes in the Tᵋ versus n Plot for SrTiO₃ Reported by Lin et al. .......... 158

10.6 Remarks .............................................. 158

Notes and References ........................................ 162

11. Some Exceptional Superconductors: La₂CuO₄ (LCO) and Heavy-fermion Superconductors (HFSCs) 163

11.1 Introduction: Features Which Make LCO and HFSCs Exceptional ................................. 163

11.1.1 LCO .............................................. 163

11.1.2 HFSCs ............................................ 165

11.1.3 Plan for this chapter ............................... 166

11.2 Basic Equations of this Chapter .............................. 167

11.3 Addressing LCO via GBCSEs ............................... 171

11.3.1 Debye temperature of La ions in LCO ............. 171

11.3.2 Dealing with LCO with the input of its Tᵋ via equations sans μ ............................. 172

11.3.3 Dealing with LCO with the input of its Tᵋ via equations sans μ and a different a value of Debye temperature ................... 172

11.3.4 LCO addressed via μ-incorporated GBCSEs: A consistency check of Eqs. (11.21) and (11.24) ........... 174
11.3.5 An explanation of the different reported values of $2|W_{20}|/k_B T_c$ for LCO based on a treatment of the number equations at $T = T_c$ and $T = 0$ in the light of the experimental findings of Tacon et al. ........................................... 175

11.3.6 Predictions of the $\mu$-incorporated GBCSEs for LCO ....................................................... 178
11.3.6.1 Values of the smaller gaps ....................... 178
11.3.6.2 Carrier concentration .......................... 178

11.4 Addressing HFSCs via BCS Equations ................ 179
11.4.1 Pairing via Ce ions in CeCoIn$_5$: Debye temperature of Ce ions ........................................ 180
11.4.2 Modification of BCS equations to deal with HFSCs ......................................................... 180
11.4.3 A consistency check of Eqs. (11.8) and (11.15) ................................................................. 181
11.4.4 Solutions of Eqs. (11.15) and (11.8) with the input of $\theta(\text{Ce}) = 73.3$ K, $T_c = 2.3$ K, and different values of $\mu$ ............................................................... 181
11.4.5 Effective mass of conduction electrons .......... 184
11.4.6 Solutions of Eqs. (11.15) and (11.8) with the input of $\theta(\text{Ce}) = 276.3$ K, $T_c = 2.3$ K, and different values of $\mu$ ............................................................... 185
11.4.7 Pairing via with the sub-lattice containing Co ions ......................................................... 186
11.4.8 Density of charge carriers .......................... 186

11.5 Remarks ................................................... 187
11.5.1 LCO: discussion ..................................... 189
11.5.2 HFSCs: discussion ................................... 189

Notes and References ....................................... 191

12.1 Introduction .............................................. 195
12.2 Solar Emission Lines .................................... 196
12.2.1 Conventional approach to identify solar emission lines ............................................. 196
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.2.2</td>
<td>The Rowland puzzle</td>
<td>196</td>
</tr>
<tr>
<td>12.2.3</td>
<td>Hot hydrogen</td>
<td>197</td>
</tr>
<tr>
<td>12.2.4</td>
<td>Equation for hot hydrogen</td>
<td>197</td>
</tr>
<tr>
<td>12.2.5</td>
<td>Eigenvalue equation for hot hydrogen for high values of T by an application of Fock’s method</td>
<td>198</td>
</tr>
<tr>
<td>12.2.6</td>
<td>Solutions of the eigenvalue equation for hot hydrogen for ( T \geq 10^5 K )</td>
<td>201</td>
</tr>
<tr>
<td>12.2.7</td>
<td>Addressing the SL 304 data</td>
<td>201</td>
</tr>
<tr>
<td>12.3</td>
<td>Quarkonium Mass Spectra</td>
<td>204</td>
</tr>
<tr>
<td>12.3.1</td>
<td>Mesons as bound states of quarks</td>
<td>204</td>
</tr>
<tr>
<td>12.3.2</td>
<td>Kernel of BSE for the ( q\bar{q} ) system for the Cornell potential</td>
<td>205</td>
</tr>
<tr>
<td>12.3.3</td>
<td>( T )-generalized equation for quarkonium</td>
<td>206</td>
</tr>
<tr>
<td>12.3.4</td>
<td>Mass spectra of the ( J/\Psi ) family</td>
<td>207</td>
</tr>
<tr>
<td>12.4</td>
<td>Remarks</td>
<td>208</td>
</tr>
<tr>
<td>12.4.1</td>
<td>Solar emission lines</td>
<td>208</td>
</tr>
<tr>
<td>12.4.2</td>
<td>Quarkonium spectra</td>
<td>211</td>
</tr>
</tbody>
</table>

Notes and References 212

Summing-Up 215

Index 221