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Constitutive modeling and experimental investigations of phase transitions in silicon under contact loading

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Thesis

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