The Fiber-Optic Gyroscope

Second Edition

Hervé C. Lefèvre
Contents

Preface to the First Edition xvi
Preface to the Second Edition xviii

CHAPTER 1
Introduction 1
References 4

CHAPTER 2
Principle of the Fiber-Optic Gyroscope 7
2.1 Sagnac Effect 7
  2.1.1 A History of Optics from Aether to Relativity 7
  2.1.2 Sagnac Effect in a Vacuum 8
  2.1.3 Sagnac Effect in a Medium 13
2.2 Active and Passive Ring Resonators 15
  2.2.1 Ring-Laser Gyroscope (RLG) 15
  2.2.2 Resonant Fiber-Optic Gyroscope (R-FOG) 18
2.3 Passive Fiber-Ring Interferometer 20
  2.3.1 Principle of the Interferometric Fiber-Optic Gyroscope (I-FOG) 20
  2.3.2 Theoretical Sensitivity of the I-FOG 22
  2.3.3 Noise, Drift, and Scale Factor 25
  2.3.4 Evaluation of Noise and Drift by Allan Variance (or Allan Deviation) 27
  2.3.5 Bandwidth 31
References 31

CHAPTER 3
Reciprocity of a Fiber Ring Interferometer 33
3.1 Principle of Reciprocity 33
  3.1.1 Single-Mode Reciprocity of Wave Propagation 33
  3.1.2 Reciprocal Behavior of a Beam Splitter 34
3.2 Minimum Configuration of a Ring Fiber Interferometer 36
  3.2.1 Reciprocal Configuration 36
  3.2.2 Reciprocal Biasing Modulation-Demodulation 38
  3.2.3 Proper (or Eigen) Frequency 41
3.3 Reciprocity with All-Guided Schemes 47
  3.3.1 Evanescent-Field Coupler (or X-Coupler or Four-Port Coupler) 47
### 3.3.2 Y-Junction

49

### 3.3.3 All-Fiber Approach

53

### 3.3.4 Hybrid Architectures with Integrated Optics: Y-Coupler Configuration

54

### 3.4 Problem of Polarization Reciprocity

58

#### 3.4.1 Rejection Requirement with Ordinary Single-Mode Fiber

58

#### 3.4.2 Use of Polarization-Maintaining (PM) Fiber

60

#### 3.4.3 Use of Depolarizer

61

#### 3.4.4 Use of an Unpolarized Source

61

### References

62

### CHAPTER 4

#### Backreflection and Backscattering

65

### 4.1 Problem of Backreflection

65

#### 4.1.1 Reduction of Backreflection with Slant Interfaces

65

#### 4.1.2 Influence of Source Coherence

67

### 4.2 Problem of Backscattering

68

#### 4.2.1 Coherent Backscattering

68

#### 4.2.2 Use of a Broadband Source

69

#### 4.2.3 Evaluation of the Residual Rayleigh Backscattering Noise

69

### References

72

### CHAPTER 5

#### Analysis of Polarization Nonreciprocities with Broadband Source and High-Birefringence Polarization-Maintaining Fiber

75

### 5.1 Depolarization Effect in High-Birefringence Polarization-Maintaining Fibers

75

### 5.2 Analysis of Polarization Nonreciprocities in a Fiber Gyroscope Using an All-Polarization-Maintaining Waveguide Configuration

77

#### 5.2.1 Intensity-Type Effects

77

#### 5.2.2 Comment About Length of Depolarization $L_d$ Versus Length of Polarization Correlation $L_{pc}$

81

#### 5.2.3 Amplitude-Type Effects

84

### 5.3 Use of a Depolarizer

87

### 5.4 Testing with Optical Coherence Domain Polarimetry (OCDP)

88

#### 5.4.1 OCDP Based on Path-Matched White-Light Interferometry

88

#### 5.4.2 OCDP Using Optical Spectrum Analysis

93

### References

93

### CHAPTER 6

#### Time Transience-Related Nonreciprocal Effects

95

### 6.1 Effect of Temperature Transience: The Shupe Effect

95

### 6.2 Symmetrical Windings

98

### 6.3 Stress-Induced T-Dot Effect

99

### 6.4 Basics of Heat Diffusion and Temporal Signature of the Shupe and T-Dot Effects

100
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>Effect of Acoustic Noise and Vibration</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>105</td>
</tr>
<tr>
<td><strong>CHAPTER 7</strong></td>
<td>Truly Nonreciprocal Effects</td>
<td>107</td>
</tr>
<tr>
<td>7.1</td>
<td>Magneto-Optic Faraday Effect</td>
<td>107</td>
</tr>
<tr>
<td>7.2</td>
<td>Transverse Magneto-Optic Effect</td>
<td>111</td>
</tr>
<tr>
<td>7.3</td>
<td>Nonlinear Kerr Effect</td>
<td>112</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>116</td>
</tr>
<tr>
<td><strong>CHAPTER 8</strong></td>
<td>Scale Factor Linearity and Accuracy</td>
<td>119</td>
</tr>
<tr>
<td>8.1</td>
<td>Problem of Scale Factor Linearity and Accuracy</td>
<td>119</td>
</tr>
<tr>
<td>8.2</td>
<td>Closed-Loop Operation Methods to Linearize the Scale Factor</td>
<td>120</td>
</tr>
<tr>
<td>8.2.1</td>
<td>Use of a Frequency Shift</td>
<td>120</td>
</tr>
<tr>
<td>8.2.2</td>
<td>Use of an Analog Phase Ramp (or Serrodyne Modulation)</td>
<td>122</td>
</tr>
<tr>
<td>8.2.3</td>
<td>Use of a Digital Phase Ramp</td>
<td>126</td>
</tr>
<tr>
<td>8.2.4</td>
<td>All-Digital Closed-Loop Processing Method</td>
<td>131</td>
</tr>
<tr>
<td>8.2.5</td>
<td>Control of the Gain of the Modulation Chain with Four-State Modulation</td>
<td>136</td>
</tr>
<tr>
<td>8.2.6</td>
<td>Potential Spurious Lock-In (or Deadband) Effect</td>
<td>139</td>
</tr>
<tr>
<td>8.3</td>
<td>Scale Factor Accuracy</td>
<td>140</td>
</tr>
<tr>
<td>8.3.1</td>
<td>Problem of Scale Factor Accuracy</td>
<td>140</td>
</tr>
<tr>
<td>8.3.2</td>
<td>Wavelength Dependence of an Interferometer Response with a Broadband Source</td>
<td>140</td>
</tr>
<tr>
<td>8.3.3</td>
<td>Effect of Phase Modulation</td>
<td>142</td>
</tr>
<tr>
<td>8.3.4</td>
<td>Wavelength Control Schemes</td>
<td>143</td>
</tr>
<tr>
<td>8.3.5</td>
<td>Mean Wavelength Change with a Parasitic Interferometer or Polarimeter</td>
<td>145</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>148</td>
</tr>
<tr>
<td><strong>CHAPTER 9</strong></td>
<td>Recapitulation of the Optimal Operating Conditions and Technologies of the I-FOG</td>
<td>151</td>
</tr>
<tr>
<td>9.1</td>
<td>Optimal Operating Conditions</td>
<td>151</td>
</tr>
<tr>
<td>9.2</td>
<td>Broadband Source</td>
<td>154</td>
</tr>
<tr>
<td>9.2.1</td>
<td>Superluminescent Diode</td>
<td>154</td>
</tr>
<tr>
<td>9.2.2</td>
<td>Rare-Earth Doped Fiber ASE Sources</td>
<td>156</td>
</tr>
<tr>
<td>9.2.3</td>
<td>Excess RIN Compensation Techniques</td>
<td>157</td>
</tr>
<tr>
<td>9.3</td>
<td>Sensing Coil</td>
<td>158</td>
</tr>
<tr>
<td>9.4</td>
<td>The Heart of the Interferometer</td>
<td>160</td>
</tr>
<tr>
<td>9.5</td>
<td>Detector and Processing Electronics</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>162</td>
</tr>
</tbody>
</table>
CHAPTER 10
Alternative Approaches for the l-FOG 165
10.1 Alternative Optical Configurations 165
10.2 Alternative Signal Processing Schemes 166
  10.2.1 Open-Loop Scheme with Use of Multiple Harmonics 166
  10.2.2 Second Harmonic Feedback 167
  10.2.3 Gated Phase Modulation Feedback 167
  10.2.4 Heterodyne and Pseudo-Heterodyne Schemes 168
  10.2.5 Beat Detection with Phase Ramp Feedback 170
  10.2.6 Dual-Phase Ramp Feedback 171
10.3 Extended Dynamic Range with Multiple Wavelength Source 171
References 172

CHAPTER 11
Resonant Fiber-Optic Gyroscope (R-FOG) 175
11.1 Principle of Operation of an All-Fiber Ring Cavity 175
11.2 Signal Processing Method 179
11.3 Reciprocity of a Ring Fiber Cavity 181
  11.3.1 Introduction 181
  11.3.2 Basic Reciprocity Within the Ring Resonator 182
  11.3.3 Excitation and Detection of Resonances in a Ring Resonator 185
11.4 Other Parasitic Effects in the R-FOG 190
Acknowledgments 192
References 193

CHAPTER 12
Conclusions 195
12.1 The State of Development and Expectations in 1993 195
12.2 The Present State of the Art, Two Decades Later 196
  12.2.1 FOG Versus RLG 196
  12.2.2 FOG Manufacturers 197
12.3 Trends for the Future and Concluding Remarks 198
Acknowledgments 198
References 199

APPENDIX A
Fundamentals of Optics for the Fiber Gyroscope 201
A.1 Basic Parameters of an Optical Wave: Wavelength, Frequency, and Power 201
A.2 Spontaneous Emission, Stimulated Emission, and Related Noises 205
  A.2.1 Fundamental Photon Noise 205
  A.2.2 Spontaneous Emission and Excess Relative Intensity Noise (Excess RIN) 206
  A.2.3 Resonant Stimulated Emission in a Laser Source 207
  A.2.4 Amplified Spontaneous Emission (ASE) 208
A.3 Propagation Equation in a Vacuum 209
A.4 State of Polarization of an Optical Wave
A.5 Propagation in a Dielectric Medium
  A.5.1 Index of Refraction
  A.5.2 Chromatic Dispersion, Group Velocity, and Group Velocity Dispersion
  A.5.3 E and B, or E and H?
A.6 Dielectric Interface
  A.6.1 Refraction, Partial Reflection, and Total Internal Reflection
  A.6.2 Dielectric Waveguidance
A.7 Geometrical Optics
  A.7.1 Rays and Phase Wavefronts
  A.7.2 Plane Mirror and Beam Splitter
  A.7.3 Lenses
A.8 Interferences
  A.8.1 Principle of Two-Wave Interferometry
  A.8.2 Most Common Two-Wave Interferometers: Michelson and Mach-Zehnder Interferometers, Young Double-Slit
  A.8.3 Channeled Spectral Response of a Two-Wave Interferometer
A.9 Multiple-Wave Interferences
  A.9.1 Fabry-Perot Interferometer
  A.9.2 Ring Resonant Cavity
  A.9.3 Multilayer Dielectric Mirror and Bragg Reflector
  A.9.4 Bulk-Optic Diffraction Grating
A.10 Diffraction
  A.10.1 Fresnel Diffraction and Fraunhofer Diffraction
  A.10.2 Knife-Edge Fresnel Diffraction
A.11 Gaussian Beam
A.12 Coherence
  A.12.1 Basics of Coherence
  A.12.2 Mathematical Derivation of Temporal Coherence
  A.12.3 The Concept of a Wave Train
  A.12.4 The Case of an Asymmetrical Spectrum
  A.12.5 The Case of Propagation in a Dispersive Medium
A.13 Birefringence
  A.13.1 Birefringence Index Difference
  A.13.2 Change of Polarization with Birefringence
  A.13.3 Interference with Birefringence
A.14 Optical Spectrum Analysis
  Reference
  Selected Bibliography

**APPENDIX B**
Fundamentals of Fiber Optics for the Fiber Gyroscope

B.1 Main Characteristics of a Single-Mode Optical Fiber
  B.1.1 Attenuation of a Silica Fiber
  B.1.2 Gaussian Profile of the Fundamental Mode
### C.1 Optical Fiber and Integrated-Optic Waveguide

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.1.2 Coupling Between an Optical Fiber and an Integrated-Optic Waveguide</td>
<td>332</td>
</tr>
<tr>
<td>C.1.3 Fundamental Mode Profile and Equivalence with an LP$_{11}$ Fiber Mode</td>
<td>333</td>
</tr>
<tr>
<td>C.1.4 Mismatch Coupling Attenuation Between a Fiber and a Waveguide</td>
<td>335</td>
</tr>
<tr>
<td>C.1.5 Low-Driving-Voltage Phase Modulator</td>
<td>336</td>
</tr>
<tr>
<td>C.1.6 Beam Splitting</td>
<td>336</td>
</tr>
<tr>
<td>C.1.7 Polarization Rejection and Birefringence-Induced Depolarization</td>
<td>338</td>
</tr>
</tbody>
</table>

### C.2 Ti-Indiffused LiNbO$_3$ Integrated Optics

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.2.1 Ti-Indiffused Channel Waveguide</td>
<td>340</td>
</tr>
<tr>
<td>C.2.2 Phase Modulation and Metallic-Overlay Polarizer with Ti-Indiffused Waveguide</td>
<td>340</td>
</tr>
</tbody>
</table>

### C.3 Proton-Exchanged LiNbO$_3$ Integrated Optics

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.3.1 Single-Polarization Propagation</td>
<td>343</td>
</tr>
<tr>
<td>C.3.2 Phase Modulation in Proton-Exchanged Waveguide</td>
<td>344</td>
</tr>
<tr>
<td>C.3.3 Theoretical Polarization Rejection of a Proton-Exchanged LiNbO$_3$ Circuit</td>
<td>345</td>
</tr>
<tr>
<td>C.3.4 Practical Polarization Rejection of Proton-Exchanged LiNbO$_3$ Circuit</td>
<td>347</td>
</tr>
<tr>
<td>C.3.5 Improved Polarization Rejection with Absorbing Grooves</td>
<td>348</td>
</tr>
<tr>
<td>C.3.6 Spurious Intensity Modulation</td>
<td>351</td>
</tr>
<tr>
<td>Selected Bibliography</td>
<td>352</td>
</tr>
</tbody>
</table>

### Appendix D

**Electromagnetic Theory of the Relativistic Sagnac Effect**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.1 Special Relativity and Electromagnetism</td>
<td>353</td>
</tr>
<tr>
<td>D.2 Electromagnetism in a Rotating Frame</td>
<td>361</td>
</tr>
<tr>
<td>D.3 Case of a Rotating Toroidal Dielectric Waveguide</td>
<td>363</td>
</tr>
<tr>
<td>Selected Bibliography</td>
<td>365</td>
</tr>
</tbody>
</table>

### Appendix E

**Basics of Inertial Navigation**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>E.1 Introduction</td>
<td>367</td>
</tr>
<tr>
<td>E.2 Inertial Sensors</td>
<td>369</td>
</tr>
<tr>
<td>E.2.1 Accelerometers (Acceleration Sensors)</td>
<td>369</td>
</tr>
<tr>
<td>E.2.2 Gyroscopes (Rotation Rate Sensors)</td>
<td>369</td>
</tr>
<tr>
<td>E.2.3 Classification of the Inertial Sensor Performance</td>
<td>370</td>
</tr>
<tr>
<td>E.3 Navigation Computation</td>
<td>370</td>
</tr>
<tr>
<td>E.3.1 A Bit of Geodesy</td>
<td>371</td>
</tr>
<tr>
<td>E.3.2 Reference Frames</td>
<td>371</td>
</tr>
<tr>
<td>E.3.3 Orientation, Velocity, and Position Computation</td>
<td>372</td>
</tr>
<tr>
<td>E.3.4 Altitude Computation</td>
<td>372</td>
</tr>
<tr>
<td>E.4 Attitude and Heading Initialization</td>
<td>373</td>
</tr>
<tr>
<td>E.4.1 Attitude Initialization</td>
<td>373</td>
</tr>
</tbody>
</table>
E.4.2  Heading Initialization with Gyrocompassing 374
E.5   Velocity and Position Initialization 375
E.6   Orders of Magnitude to Remember 375
Selected Bibliography 376

List of Abbreviations 377
List of Symbols 379
About the Author 385
Index 387