Crystal Symmetry, Lattice Vibrations, and Optical Spectroscopy of Solids
A Group Theoretical Approach

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