Moduli Spaces of Riemann Surfaces

Benson Farb
Richard Hain
Eduard Looijenga
Editors
Contents

Preface ix

Benson Farb, Richard Hain, and Eduard Looijenga
Introduction 1

Yair N. Minsky
A Brief Introduction to Mapping Class Groups 5
1. Definitions, examples, basic structure 7
2. Hyperbolic geometry, laminations and foliations 19
3. The Nielsen-Thurston classification theorem 28
4. Classification continued, and consequences 35
5. Further reading and current events 39
Bibliography 41

Ursula Hamenstädt
Teichmüller Theory 45
Introduction 47
Lecture 1. Hyperbolic surfaces 49
Lecture 2. Quasiconformal maps 63
Lecture 3. Complex structures, Jacobians and the Weil Petersson form 75
Lecture 4. The curve graph and the augmented Teichmüller space 85
Lecture 5. Geometry and dynamics of moduli space 93
Bibliography 107

Nathalie Wahl
The Mumford Conjecture, Madsen-Weiss and Homological Stability
for Mapping Class Groups of Surfaces 109
Introduction 111
Lecture 1. The Mumford conjecture and the Madsen-Weiss theorem 113
1. The Mumford conjecture 113
2. Moduli space, mapping class groups and diffeomorphism groups 113
3. The Mumford-Morita-Miller classes 115
CONTENTS

Andrew Putman
The Torelli Group and Congruence Subgroups of the Mapping Class Group 169
   Introduction 171

Lecture 1.  The Torelli group 173
Lecture 2.  The Johnson homomorphism 179
Lecture 3.  The abelianization of Mod_{g,n}(p) 185
Lecture 4.  The second rational homology group of Mod_{g}(p) 189

Bibliography 195

Carel Faber
Tautological Algebras of Moduli Spaces of Curves 197
   Introduction 199

Lecture 1.  The tautological ring of M_{g} 201
   Exercises 208

Lecture 2.  The tautological rings of \overline{M}_{g,n} and of some natural partial compactifications of M_{g,n} 211
   Exercises 215

Bibliography 217

Scott A. Wolpert
Mirzakhani’s Volume Recursion and Approach for the Witten-Kontsevich Theorem on Moduli Tautological Intersection Numbers 221
   Prelude 225

Lecture 1.  The background and overview 231
Lecture 2.  The McShane-Mirzakhani identity 239
Lecture 3.  The covolume formula and recursion 243
Lecture 4.  Symplectic reduction, principal S^{1} bundles and the normal form 249
Lecture 5.  The pattern of intersection numbers and Witten-Kontsevich 255

Questions for the problem sessions 261

Bibliography 265

Martin Möller
Teichmüller Curves, Mainly from the Viewpoint of Algebraic Geometry 267

1.  Introduction 269
2. Flat surfaces and $SL_2(\mathbb{R})$-action  
   2.1. Flat surfaces and translation structures  
   2.2. Affine groups and the trace field  
   2.3. Strata of $\Omega M_g$ and hyperelliptic loci  
   2.4. Spin structures and connected components of strata  
   2.5. Stable differentials and Deligne-Mumford compactification

3. Curves and divisors in $M_g$  
   3.1. Curves and fibered surfaces  
   3.2. Picard groups of moduli spaces  
   3.3. Special divisors on moduli spaces  
   3.4. Slopes of divisors and of curves in $M_g$

4. Variation of Hodge structures and real multiplication  
   4.1. Hilbert modular varieties and the locus of real multiplication  
   4.2. Examples

5. Teichmüller curves  
   5.1. Square-tiled surfaces and primitivity  
   5.2. The VHS of $T$ curves  
   5.3. Proof of the VHS decomposition and real multiplication  
   5.4. Cusps and sections of $T$ curves  
   5.5. The classification problem of $T$ curves: state of the art

6. Lyapunov exponents  
   6.1. Motivation: Asymptotic cycles, deviations and the wind-tree model  
   6.2. Lyapunov exponents  
   6.3. Lyapunov exponents for Teichmüller curves  
   6.4. Non-varying properties for sums of Lyapunov exponents  
   6.5. Lyapunov exponents for general curves in $M_g$ and in $A_g$  
   6.6. Known results and open problems

Bibliography

Makoto Matsumoto

Introduction to arithmetic mapping class groups

Introduction

Lecture 1. Algebraic fundamental groups  
Lecture 2. Monodromy representation on fundamental groups  
Lecture 3. Arithmetic mapping class groups  
Lecture 4. Topology versus arithmetic  
Lecture 5. The conjectures of Oda and Deligne-Ihara

APPENDIX: Algebraic fundamental groups via fiber functors

Bibliography