Enhancing Mathematics Understanding through Visualization:
The Role of Dynamical Software

Samer Habre
Lebanese American University, Lebanon
Section 1
The Theory

Chapter 1
Technology and Differential Equations .......................................................... 1

*John Hubbard, Cornell University, USA*

Differential equations are the main way in which we make mathematical models of the real world. This is true in just about all fields, from physics to fluid mechanics, from astronomy to finance: if you want to understand how something evolves, or if you want to understand equilibria, you will need to set up and “solve” differential equations. For instance, our understanding of global warming depends mainly on analyzing the differential equations modeling the weather and seeing how their behavior depends on the concentration of greenhouse gasses in the atmosphere. The future of humanity depends on our getting it right.

Chapter 2
Sometimes Less is More: Examples of Student-Centered Technology as Boundary Objects in Differential Equations ......................................................... 12

*Karen Keene, North Carolina State University, USA*

*Chris Rasmussen, San Diego State University, USA*

As described in the communities of practice literature (Lave & Wenger, 1991; Wenger, 1998), boundary objects are material things that interface two or more communities of practice. Extending this, Hoyles, Noss, Kent, and Bakker (2010) defined technology-enhanced boundary objects as, “software tools that adapt or extend symbolic artefacts identified from existing work practice, that are intended
to act as boundary objects, for the purposes of employees' learning and enhancing workplace communication" (p. 17). The authors adapt this idea to the undergraduate mathematics classroom and use the phrase “classroom technology-enhanced boundary object” to refer to a piece of software that acts as a boundary object between the classroom community and the mathematical community. They provide three extended examples of these objects as used in a first semester differential equations classroom to illustrate how students' mathematical activity may advance as they interact with the software. These examples show how the applets operate to provide a way for the classroom community to implicitly encounter the mathematical community through the authentic practices of mathematics (Rasmussen, Zandieh, King, & Teppo, 2005). The first example centers on students beginning experience with a tangent vector field applet. The second example develops as the students learn more about solutions to differential equations and leads to a statement of the uniqueness theorem. In the third example, students use a specially designed applet that creates a numerical approximation and its associated image in 3-space relating to a non-technological visualization task that introduces solutions to systems of differential equations.

Chapter 3
"Click, Drag, Think!" Posing and Exploring Conjectures with Dynamic Geometry Software ................................................................. 37

Thomas Gawlick, Leibniz Universität Hannover, Germany

The author points out that to fully exploit the heuristic potential of Dynamic Geometry Software (DGS) and to increase the heuristic literacy of students, extant DGS teaching units have to be ameliorated in several ways. Thus the author develops a twofold conceptual framework: heuristic reconstruction and heuristic instrumentation of problems. Its origin is rooted in the literature, its use is demonstrated by various examples, and its value is made plausible by an introductory teaching unit and an advanced case study.

Chapter 4
Dynamical Mathematical Software: Tools for Learning and for Research .......... 70

Samer Habre, Lebanese American University, Lebanon

Understanding mathematical concepts is many-folded. Traditional mathematics mostly emphasizes the algebraic/analytical aspect of a problem with minimal reference to its graphical aspect and/or numerical one. In a modern learning environment, however, multiple representations of concepts are proving to be essential for the teaching of mathematics. The availability of user-friendly dynamical software programs has paved the way for a radical yet smooth way for changing the way
mathematical concepts are perceived. This chapter presents some of the author’s attempts for employing innovative methods in teaching topics in calculus, in differential and difference equations. The focus is on the use of dynamical programs that boost the visual component of the topics being investigated, hence contributing to a more complete understanding of these topics.

Chapter 5
Nonlinear is Essential, Linearization is Not Enough, Visualization is Absolutely Necessary

Beverly West, Cornell University, USA

Linear differential equations have been well understood for some time and are an important tool for studying the nonlinear systems that most frequently arise in mathematical models of real world systems. Nonlinear systems do not usually have formula solutions, but with graphics, we can see the behaviors of the solutions and thereby “understand” the differential equations. Dynamic and interactive presentations provide students with a streamlined route to understanding these behaviors, resulting in immense power and efficiency that was not previously available at the undergraduate level.

Chapter 6
Vectors and Differential Equations: A Visual Approach using Autograph

Douglas Butler, iCT Training Centre, UK

This chapter is inspired by a session the author gave at the 10th EMAC Conference (Engineering, Mathematics, and Applications) at the University of Technology, Sydney, in December 2011. The audience was university teachers, but the software, Autograph, was designed for use in High Schools. The author was able to show how a simple, pedagogically focused interface could be used to create a highly visual approach to the teaching of two favourite topics: Vectors (in 2D and 3D) and Differential Equations (1st and 2nd Order).

Chapter 7
Interactive Applets in Calculus and Engineering Courses

Heidi Burgiel, Bridgewater State University, USA

Chad Lieberman, MIT, USA

Haynes Miller, MIT, USA

Karen Willcox, MIT, USA

This chapter reports on the use of specialized computer applets (“Mathlets”) in two different contexts: on-line instruction in calculus through MIT OpenCourseWare and on-campus laboratory exercises on stability of difference schemes in class. Specific applets are described. Three use-cases with varying levels of sophistication (elementary, intermediate, and advanced) are outlined.
Chapter 8
Applets for Mathematical Learning.................. 145

Robert Terrell, Cornell University, USA

The purpose of this chapter is to describe dynamic interactive applets, which are available to all students, to assist in the understanding of sophomore-level engineering mathematics. The topics illustrated here with screen shots are the partial differential equations for heat conduction in one and two space dimensions, the wave equation and Laplace equation in two space dimensions, systems of three ordinary differential equations, and the display of vector fields in three dimensions. An applet is also shown for iterated function systems, although this is not usually part of the engineering sequence. Additional applets mentioned here and available online include the wave equation in one space dimension and two-by-two matrices. The purpose of these applets, all written by the author and freely available, is to help students experiment visually and dynamically with the mathematical concepts.

Chapter 9
Dynamical Software and the Derivative Concept........ 153

Ljubica Dikovic, Business Technical University, Serbia

Modern teaching trends impose the need of spending less time on the manipulative approach to differential and integral calculus, putting the accent on the conceptual understanding of the subject. This chapter presents the standard approach and method used to teach the derivative of a function and indicates some critical points in the teaching of the derivative, offering, at the same time, suggestions for overcoming them. As a supplement, the author gives e-resources that can make possible the implementation of a stimulating, visual, dynamic, and broadened method for teaching the derivative of a function.

Chapter 10
Supporting the Development of College-Level Students’ Conceptions of Statistical Inference.................. 167

Maria Meletiou-Mavrotheris, European University Cyprus, Cyprus

The transition from descriptive to inferential statistics is a known area of difficulty for students taking introductory statistics courses. This chapter shares the experiences from a teaching experiment in a college-level introductory statistics classroom that implemented an informal, data-driven approach to statistical inference using the dynamic statistics software Fathom© as an investigation tool. Findings from the study indicate that the informal inferences on which instruction focused in the first part of the course helped students develop understandings of fundamental aspects of inferential and argumentative reasoning that served as foundations for the formal study of inferential statistics in the latter part of the course. The affordances offered by the tool for delving deeply into the data to make sense of the situation at hand were instrumental in supporting student understanding of both informal and formal inferential statistics.
Section 3
Outside The Norm

Chapter 11
Coping with Infinity: Using TI-Nspire\textsuperscript{TM} CAS to Bring Alive Multiple
Representations in Mathematics .............................................................................201
\textit{Bjorn Felsager, Midtsjællands Gymnasieskoler, Denmark}

This chapter outlines the structure of the initial module of a pilot project investigating
the benefits of using dynamical mathematical software when teaching students about
cross-curricular models of thinking. Focus is on the use of conceptual metaphors
when teaching the concept of infinity in mathematics as well as in literature. The use
of dynamical mathematics software facilitated constructions of multiple representa¬
tions in problem solving in a series of carefully selected problems involving infinity.
The pilot project is currently being evaluated, but preliminary findings indicate an
increased student awareness of the usefulness of the software when learning how to
use mathematical inscriptions for problem solving. The pilot project will be expanded
in the Fall 2011 to include 6 classes across Denmark to test the applicability of the
software, the teaching model and the teaching materials in a variety of classes to
see if the initial successes of the project can be replicated.

Chapter 12
String Art and Linear Iterative Systems ..................................................................212
\textit{Samer Habre, Lebanese American University, Lebanon}

String art dates back to the 19th century when it was initially invented to ease the
delivery of some mathematical ideas. Since then, the art has evolved and so has its
use in mathematics. In this chapter, the authors see how some of the phase portraits
for 2 x 2 linear homogeneous iterative systems exhibit some artistic behavior that
resembles this form of art. The investigation gives a sufficient condition for the
solutions of such systems to form closed cycles. However, in other situations the
cycles formed are infinite, producing some fascinating examples of string art.

Related References ..................................................................................................222

Compilation of References .....................................................................................255

About the Contributors ............................................................................................268

Index .........................................................................................................................273