An Introduction to Non-Perturbative Foundations of Quantum Field Theory

Franco Strocchi
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   2 Relativistic Schrödinger wave mechanics
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      2.3 Dirac equation
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   3 Relativistic particle interactions and quantum mechanics
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