

**Shape Optimization for Fluids Using T-Splines for
Shape Representation and Stabilized Finite Elements
for the Fluid Flow Simulations**

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