An Invitation to $q$-Series
From Jacobi's Triple Product Identity to Ramanujan's "Most Beautiful Identity"

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