CONTENTS

Preface xix

1. Basic concepts 1
   1.1 Introduction 1
   1.2 Prestressed concrete 1
   1.3 Economics of prestressed concrete 4

2. Technology of prestressing 5
   2.1 Methods of prestressing 5
   2.2 Pre-tensioning 5
      2.2.1 Debonding/blanketing of strands 10
      2.2.2 Deflecting/draping/harping of strands 11
      2.2.3 Loss of prestress at transfer 12
      2.2.4 Transmission length 12
         2.2.4.1 Example of calculation of transmission length 14
   2.3 Post-tensioning 15
      2.3.1 Post-tensioning anchors 18
      2.3.2 Loss of prestress at transfer 21
      2.3.3 External prestressing 21
      2.3.4 Unbonded systems 22

3. Material properties 25
   3.1 Properties of concrete 25
   3.2 Compressive strength of concrete 25
   3.3 Tensile strength of concrete 26
   3.4 Deformational properties 27
      3.4.1 Elastic moduli 27
      3.4.2 Creep coefficient 27
      3.4.3 Shrinkage 31
   3.5 Stress-strain relationship 33
      3.5.1 Parabolic-rectangular relationship 34
      3.5.2 Bi-linear relationship 35
      3.5.3 Confined concrete 35
   3.6 Permissible stresses in concrete 36
   3.7 Prestressing steel 37
   3.8 Relaxation 39
   3.9 Maximum Stress at Jacking 40
   3.10 Long-term loss of prestress 40
   3.11 References to Eurocode 2 clauses 40
4. **Serviceability limit state design of pre-tensioned beams** 43
   4.1 Design of prestressed concrete structure 43
   4.2 Beam design based on engineers' theory of bending 43
      4.2.1 Sign convention 44
      4.2.2 Example of beam designed based on engineer's theory of bending 44
   4.3 Development of SLS design equations 47
      4.3.1 Example of SLS design equations 49
      4.3.2 Magnel diagram 50
      4.3.3 Choice of prestress and eccentricity 52
      4.3.4 Stress check 53
      4.3.5 Debonding 54
      4.3.6 Choice of prestress and eccentricity at different sections 56
   4.4 Initial sizing of section 56
      4.4.1 Example of preliminary sizing 58
   4.5 Composite beam section 59
      4.5.1 Magnel equations for composite beam 60
      4.5.2 Shrinkage stress calculation 61
      4.5.3 Example of shrinkage stress calculation 64
      4.5.4 Magnel diagrams for a composite beam 67
      4.5.5 Choice of prestress and eccentricity at different sections 70
   4.6 Cracking 71
   4.7 Thermal stress calculation 71
      4.7.1 Heating 72
      4.7.2 Cooling 73
      4.7.3 Calculation of stresses due to thermal gradients 73
      4.7.4 Example of thermal stress calculation 74
         4.7.4.1 Thermal stress calculation: Heating 75
         4.7.4.2 Thermal stress calculation: Cooling 78
   4.8 Detailing 79
   4.9 References to Eurocode 1 and Eurocode 2 clauses 80

5. **Bonded post-tensioned structures** 81
   5.1 Post-tensioned beams 81
   5.2 Cable profile in a post-tensioned beam 81
      5.2.1 Example of permitted cable zone 82
         5.2.1.1 Magnel equations 85
         5.2.1.2 Determination of maximum eccentricity 85
         5.2.1.3 Determination of cable zone 87
         5.2.1.4 Detailing of post-tensioned tendons 88
   5.3 Concept of equivalent loads 88
      5.3.1 General equation for equivalent loads 90
      5.3.2 General equation for distributed loads for a parabolic profile 91
6. **Statically indeterminate post-tensioned structures** 95

6.1 Introduction 95
6.1.1 Primary and secondary moments 95
6.1.2 Prestressing of a propped cantilever 97
6.2 Analysis to determine the moment distribution due to prestress 98
6.2.1 Equivalent loads for a cable profile of a single parabola 99
6.2.2 General equation for equivalent loads for a cable profile consisting of three parabolic segments 100
6.2.3 General equation for equivalent loads for a cable profile consisting of four parabolic segments 105
6.2.3.1 Alternative profile consisting of three parabolas instead of four parabolas 111
6.2.3.2 Alternative profile consisting of two parabolas instead of three parabolas 114
6.2.4 Loss of prestress and equivalent loads 116
6.3 Fixed end moments 116
6.3.1 Fixed end moments for three-parabola cable profile 117
6.3.2 Fixed end moments for four-parabola cable profile 118
6.4 Analysis of a continuous beam for moment distribution due to prestress 118
6.4.1 Distribution of shear force 121
6.5 Cable profile consisting of linear variation between supports 122
6.6 Determination of prestress and cable profile:
   - Example of a continuous bridge beam 122
   - 6.6.1 Analysis of the bridge 123
   - 6.6.2 Determination of prestress and eccentricity 126
   - 6.6.3 Refined analysis due to equivalent loads 128
     - 6.6.3.1 Fixed end moments for three parabola cable profile 129
     - 6.6.3.2 Fixed end moments for four parabola cable profile 129
     - 6.6.3.3 Moments at supports for the cable profile 129
     - 6.6.3.4 Choice of prestress at service 129
     - 6.6.3.5 Stress check at transfer and service 132
6.7 Concordant cable profile 132
6.8 Choice of tendon size and location of tendons 132
6.9 Equivalent loads and variable second moment of area 134
   - 6.9.1 Shift in the centroidal axis in box girders 136
6.10 Equivalent loads and variable second moment of area 136
6.11 Thermal stress analysis and continuous structures 139
# Prestressed Concrete Design

6.11.1 Thermal stress calculation: heating 139  
6.11.2 Thermal stress calculation: cooling 142  
6.12 Reduction of moment over support in continuous beams 144  
6.13 References to Eurocode 2 clauses 145

## 7. Ultimate bending strength calculations 147

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Introduction</td>
<td>147</td>
</tr>
<tr>
<td>7.2</td>
<td>Stress distribution at different stages of loading</td>
<td>147</td>
</tr>
<tr>
<td>7.3</td>
<td>Stress - strain relationship for concrete</td>
<td>149</td>
</tr>
<tr>
<td>7.4</td>
<td>Rectangular stress block in bending strength calculations</td>
<td>149</td>
</tr>
<tr>
<td>7.5</td>
<td>Stress - strain relationship for steel</td>
<td>150</td>
</tr>
<tr>
<td>7.6</td>
<td>Strain and stress in steel</td>
<td>150</td>
</tr>
<tr>
<td>7.6.1</td>
<td>Prestress and pre-strain in steel</td>
<td>150</td>
</tr>
<tr>
<td>7.6.2</td>
<td>Strain due to bending in steel</td>
<td>151</td>
</tr>
<tr>
<td>7.6.3</td>
<td>Total strain and stress in steel</td>
<td>151</td>
</tr>
<tr>
<td>7.7</td>
<td>The strain compatibility method</td>
<td>151</td>
</tr>
<tr>
<td>7.8</td>
<td>Properties of a trapezium</td>
<td>152</td>
</tr>
<tr>
<td>7.9</td>
<td>Ultimate moment calculation of a bridge beam</td>
<td>152</td>
</tr>
<tr>
<td>7.10</td>
<td>Ultimate moment calculation of a composite bridge beam</td>
<td>157</td>
</tr>
<tr>
<td>7.11</td>
<td>Use of additional unstressed steel</td>
<td>162</td>
</tr>
<tr>
<td>7.12</td>
<td>Stress-strain relationship for unstressed reinforcing steel</td>
<td>163</td>
</tr>
<tr>
<td>7.13</td>
<td>Example of ultimate moment calculation with stressed and unstressed steels</td>
<td>164</td>
</tr>
<tr>
<td>7.14</td>
<td>Calculation of $M_u$ using tabular values</td>
<td>167</td>
</tr>
<tr>
<td>7.15</td>
<td>Calculation of $M_u$ for statically indeterminate beams</td>
<td>169</td>
</tr>
<tr>
<td>7.16</td>
<td>Reference to Eurocode 2 clauses</td>
<td>170</td>
</tr>
</tbody>
</table>

## 8. Analysis of cracked sections 171

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>Introduction</td>
<td>171</td>
</tr>
<tr>
<td>8.2</td>
<td>Cracked section analysis</td>
<td>171</td>
</tr>
<tr>
<td>8.3</td>
<td>Cracked section analysis of a double T-beam</td>
<td>172</td>
</tr>
<tr>
<td>8.3.1</td>
<td>Stress - strain relationship for concrete</td>
<td>174</td>
</tr>
<tr>
<td>8.3.2</td>
<td>Stress - strain relationship for steel</td>
<td>174</td>
</tr>
<tr>
<td>8.3.3</td>
<td>Cracked section analysis</td>
<td>174</td>
</tr>
<tr>
<td>8.4</td>
<td>Partially prestressed beam</td>
<td>180</td>
</tr>
<tr>
<td>8.5</td>
<td>Composite beam</td>
<td>183</td>
</tr>
<tr>
<td>8.5.1</td>
<td>Magnel diagram for composite beam</td>
<td>184</td>
</tr>
</tbody>
</table>

## 9. Ultimate shear and torsional strength calculations 193

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.1</td>
<td>Introduction</td>
<td>193</td>
</tr>
<tr>
<td>9.2</td>
<td>Shear capacity of a section without shear reinforcement and uncracked in flexure</td>
<td>195</td>
</tr>
</tbody>
</table>
### 11.2 Elastic loss in pre-tensioned beams

- 11.2.1 Example of elastic loss calculation

### 11.2.2 Elastic loss in post-tensioned beams

### 11.2.3 Loss of prestress due to friction and wobble

- 11.2.3.1 Derivation of loss of prestress due to friction
- 11.2.3.2 Example of calculation of loss of prestress due to friction and wobble
- 11.2.3.3 Calculation of $\theta$ for different profiles

### 11.2.4 Loss due to draw-in of wedges

- 11.2.4.1 Example of loss of prestress due to draw-in

### 11.3 Loss of prestress due to creep, shrinkage and relaxation

- 11.3.1 Example of final loss calculation

### 11.4 References to Eurocode 2 clauses

### 12. Design of slabs

- 12.1 Introduction
- 12.2 Typical beam and slab depths
  - 12.2.1 Effective span of slabs for different support conditions
- 12.3 One-way spanning slabs
  - 12.3.1 Design of a one-way spanning slab
  - 12.3.2 Analysis for applied loading
  - 12.3.3 Choice of prestress
  - 12.3.4 Calculation of losses
  - 12.3.5 Calculation of correct equivalent loads
  - 12.3.6 Calculation of moment distribution at service
  - 12.3.7 Calculation of stress distribution at service
  - 12.3.8 Calculation of stress distribution at transfer
- 12.4 Edge-supported two-way spanning slab
  - 12.4.1 Design of a two-way spanning slab
- 12.5 Flat slabs
- 12.6 Methods of analysis of flat slabs
- 12.7 Example of the design of flat slab
- 12.8 Finite element analysis of flat slab
  - 12.8.1 Results of analysis for dead load
  - 12.8.2 Results of analysis for dead plus live load pattern 1
  - 12.8.3 Results of analysis for dead plus live load pattern 2
  - 12.8.4 Results of analysis for dead plus live load pattern 3
  - 12.8.5 Results of analysis for dead plus live load pattern 4
- 12.9 Finite element analysis of a strip of flat slab
  - 12.9.1 Results of analysis for dead load
  - 12.9.2 Results of analysis for dead plus live load pattern 1
  - 12.9.3 Results of analysis for dead plus live load pattern 2
  - 12.9.4 Results of analysis for dead plus live load pattern 3
  - 12.9.5 Results of analysis for dead plus live load pattern 4
### 14. Loading on buildings

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.1 Introduction</td>
<td>377</td>
</tr>
<tr>
<td>14.2 Limit states</td>
<td>378</td>
</tr>
<tr>
<td>14.3 Classification of actions</td>
<td>379</td>
</tr>
<tr>
<td>14.4 Characteristic values of actions</td>
<td>379</td>
</tr>
<tr>
<td>14.5 Design values of actions</td>
<td>380</td>
</tr>
<tr>
<td>14.6 Combination of actions</td>
<td>381</td>
</tr>
<tr>
<td>14.6.1 Combination of actions for ULS</td>
<td>381</td>
</tr>
<tr>
<td>14.6.2 Values of ( \gamma ) factors</td>
<td>382</td>
</tr>
<tr>
<td>14.6.3 Examples of the use of ( \gamma ) factors</td>
<td>383</td>
</tr>
<tr>
<td>14.7 Combination of actions for SLS</td>
<td>389</td>
</tr>
<tr>
<td>14.8 References to Eurocode 1 clauses</td>
<td>390</td>
</tr>
</tbody>
</table>

### 15. Loading on bridges

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.1 Introduction</td>
<td>393</td>
</tr>
<tr>
<td>15.2 Notional Lanes</td>
<td>393</td>
</tr>
<tr>
<td>15.3 Load models</td>
<td>394</td>
</tr>
<tr>
<td>15.3.1 Load Model 1</td>
<td>394</td>
</tr>
<tr>
<td>15.3.2 Load Model 2</td>
<td>395</td>
</tr>
<tr>
<td>15.3.3 Load Model 3</td>
<td>395</td>
</tr>
<tr>
<td>15.3.4 Load Model 4</td>
<td>396</td>
</tr>
<tr>
<td>15.4 Dispersal of concentrated load</td>
<td>396</td>
</tr>
<tr>
<td>15.5 Horizontal forces</td>
<td>396</td>
</tr>
<tr>
<td>15.5.1 Breaking forces</td>
<td>396</td>
</tr>
<tr>
<td>15.5.2 Centrifugal forces</td>
<td>397</td>
</tr>
<tr>
<td>15.6 Loads on footways, cycle tracks and foot bridges</td>
<td>397</td>
</tr>
<tr>
<td>15.7 Groups of traffic loads</td>
<td>398</td>
</tr>
<tr>
<td>15.8 Combinations of actions for ULS</td>
<td>398</td>
</tr>
<tr>
<td>15.9 Values of ( \gamma ) factors</td>
<td>399</td>
</tr>
<tr>
<td>15.10 Values of ( \psi ) factors for road bridges</td>
<td>399</td>
</tr>
<tr>
<td>15.11 Combinations of actions for SLS</td>
<td>399</td>
</tr>
<tr>
<td>15.12 References to Eurocode 1 clauses</td>
<td>400</td>
</tr>
</tbody>
</table>
16. Analysis and design of bridge decks 401
16.1 Introduction 401
16.1.1 Balanced Cantilever Construction 402
16.2 Methods of analysis 405
16.3 Grillage analysis 407
16.3.1 Aspects of behaviour ignored in grillage analysis 407
16.3.2 Edge stiffening 409
16.4 Torsional constant 409
16.4.1 Torsional constant of solid sections 411
16.4.2 Torsional constant of thin-walled closed hollow sections 411
16.5 Example of analysis of a beam and slab deck 412
16.5.1 Bending properties of precast beam 412
16.5.2 Section properties of interior composite beam 414
16.5.3 Section properties of end composite beam 416
16.5.4 Torsion constant for composite beam 416
16.5.5 Alternative expressions for approximate value of J for rectangular cross sections 418
16.5.6 Section properties of transverse beams 418
16.5.7 Material properties 419
16.5.8 Calculation of live loads and bending moment distribution in beam elements: SLS 419
16.6 Stresses due to shrinkage of slab 426
16.7 Thermal stresses in the composite beam 426
16.7.1 Thermal stresses: heating 427
16.7.2 Thermal stresses: cooling 429
16.8 Stress distribution at SLS due to external loads 432
16.9 Magnel diagrams 433
16.9.1 Stress checks 437
16.10 Calculation of live loads and bending moment distribution in beam elements: ULS 438
16.11 Self-weight moments 443
16.12 Ultimate moment capacity: Mid-span section 443
16.13 Ultimate shear force 447
16.13.1 Analysis to determine maximum shear force along the span: Cases 1 – 4 448
16.13.2 Analysis to determine maximum shear force along the span: Cases 5 – 8 452
16.13.3 Summary of results 453
16.13.4 Design of shear reinforcement 454
16.14 Design of a post-tensioned box girder bridge 459
16.14.1 Calculation of moments at SLS 461
16.14.2 Thermal stresses: Heating 464
16.14.3 Thermal stresses: Cooling 466
16.14.4 Determination of prestress and eccentricity 467
16.14.5 Stress calculation at SLS 471
16.14.6 Calculation of moments at ULS 474
16.14.7 Calculation of moment capacity at ULS 477
16.14.8 Calculation of shear force at ULS 479
16.14.9 Calculation of twisting moment at ULS 481
16.14.10 Design of shear and torsional reinforcement 483
16.14.11 Longitudinal reinforcement to resist torsion 487
16.14.12 Stress analysis of the deck 487
16.15 Eurocode 2 rules for reinforcement at anchorages 489
16.16 External and internal tendons: A comparison 491
16.17 References to Eurocode 2 clauses 491

17. Lower bound approaches to design at ultimate limit state 493

17.1 Introduction 493
17.2 Theory of Plasticity 493
17.3 In-plane stresses 494
17.3.1 Examples of reinforcement calculations 497
17.3.2 Presence of prestressing cables 503
17.4 Designs for a combination of in-plane and flexural forces 504
17.4.1 Example of design for a combination of in-plane and flexural forces 507
17.5 Criterion for cracking 508
17.6 Out-of-plane shear 510
17.7 Strut and tie method of design 511
17.7.1 B and D Regions 511
17.7.2 Saint Venant's principle 512
17.7.3 An example of strut-tie modelling 514
17.7.4 Design of struts 515
17.7.5 Types of nodal zones 517
17.7.6 Correct layout of struts and ties 520
17.7.6.1 Correct layout of struts and ties: deep beam 520
17.7.6.2 Correct layout of struts and ties: corbel 521
17.7.6.2.1 Code recommendation for design of corbel 523
17.7.6.3 Correct layout of struts and ties: half-joint 525
17.7.6.4 Correct layout of struts and ties: end-block 527
17.7.6.5 Reinforcement at frame corners 529
17.8 Reference to Eurocode 2 clauses 531

18. Design for earthquake resistance 533
18.1 Introduction 533
18.2 Ductility 535
18.3 Types of structural systems 536
19.4 Determination of prestress 575
19.5 Cracking moment 578
19.6 Ultimate moment capacity 579
19.7 Ultimate shear capacity 580
19.8 Calculation of deflection 582
19.9 References to Eurocode 2 clauses 585

20. References 587

Index 591