Contents

Preface xv

How to Use This Book xix

Chapter 1 Logical Thinking 1
1.1 Formal Logic 2
   1.1.1 Connectives and Propositions 2
   1.1.2 Truth Tables 4
   1.1.3 Logical Equivalences 6
Exercises 1.1 9
1.2 Propositional Logic 15
   1.2.1 Tautologies and Contradictions 16
   1.2.2 Derivation Rules 17
   1.2.3 Proof Sequences 19
   1.2.4 Forward—Backward 21
Exercises 1.2 22
1.3 Predicate Logic 27
   1.3.1 Predicates 27
   1.3.2 Quantifiers 28
   1.3.3 Translation 29
   1.3.4 Negation 31
   1.3.5 Two Common Constructions 33
Exercises 1.3 34
1.4 Logic in Mathematics 41
   1.4.1 The Role of Definitions in Mathematics 41
   1.4.2 Other Types of Mathematical Statements 43
   1.4.3 Counterexamples 44
   1.4.4 Axiomatic Systems 45
Exercises 1.4 49
Chapter 2  Relational Thinking  65  

2.1 Graphs  66  
   2.1.1 Edges and Vertices  66  
   2.1.2 Terminology  67  
   2.1.3 Modeling Relationships with Graphs  69  
Exercises 2.1  74  

2.2 Sets  80  
   2.2.1 Membership and Containment  81  
   2.2.2 New Sets from Old  82  
   2.2.3 Identities  86  
Exercises 2.2  88  

2.3 Functions  92  
   2.3.1 Definition and Examples  92  
   2.3.2 One-to-One and Onto Functions  96  
   2.3.3 New Functions from Old  99  
Exercises 2.3  101  

2.4 Relations and Equivalences  106  
   2.4.1 Definition and Examples  106  
   2.4.2 Graphs of Relations  107  
   2.4.3 Relations vs. Functions  108  
   2.4.4 Equivalence Relations  109  
   2.4.5 Modular Arithmetic  112  
Exercises 2.4  115  

2.5 Partial Orderings  119  
   2.5.1 Definition and Examples  119  
   2.5.2 Hasse Diagrams  120  
   2.5.3 Topological Sorting  122  
   2.5.4 Isomorphisms  124  
   2.5.5 Boolean Algebras  127  
Exercises 2.5  129  

2.6 Graph Theory  134  
   2.6.1 Graphs: Formal Definitions  134  
   2.6.2 Isomorphisms of Graphs  135  

1.5 Methods of Proof  53  
   1.5.1 Direct Proofs  54  
   1.5.2 Proof by Contraposition  56  
   1.5.3 Proof by Contradiction  58  
Exercises 1.5  60
Chapter 3 Recursive Thinking 149

3.1 Recurrence Relations 150
  3.1.1 Definition and Examples 150
  3.1.2 The Fibonacci Sequence 151
  3.1.3 Modeling with Recurrence Relations 152

Exercises 3.1 156

3.2 Closed-Form Solutions and Induction 161
  3.2.1 Guessing a Closed-Form Solution 161
  3.2.2 Polynomial Sequences: Using Differences‡ 163
  3.2.3 Inductively Verifying a Solution 164

Exercises 3.2 169

3.3 Recursive Definitions 172
  3.3.1 Definition and Examples 173
  3.3.2 Writing Recursive Definitions 176
  3.3.3 Recursive Geometry 178
  3.3.4 Recursive Jokes 181

Exercises 3.3 182

3.4 Proof by Induction 187
  3.4.1 The Principle of Induction 188
  3.4.2 Examples 189
  3.4.3 Strong Induction 193
  3.4.4 Structural Induction 196

Exercises 3.4 198

3.5 Recursive Data Structures 202
  3.5.1 Lists 202
  3.5.2 Efficiency 207
  3.5.3 Binary Search Trees Revisited 208

Exercises 3.5 209

Chapter 4 Quantitative Thinking 215

4.1 Basic Counting Techniques 216
  4.1.1 Addition 216
  4.1.2 Multiplication 217
4.1.3 Mixing Addition and Multiplication 221
Exercises 4.1 223
4.2 Selections and Arrangements 227
  4.2.1 Permutations: The Arrangement Principle 227
  4.2.2 Combinations: The Selection Principle 230
  4.2.3 The Binomial Theorem† 233
Exercises 4.2 235
4.3 Counting with Functions 240
  4.3.1 One-to-One Correspondences 240
  4.3.2 The Pigeonhole Principle 243
  4.3.3 The Generalized Pigeonhole Principle 244
  4.3.4 Ramsey Theory‡ 246
Exercises 4.3 246
4.4 Discrete Probability 252
  4.4.1 Definitions and Examples 253
  4.4.2 Applications 254
  4.4.3 Expected Value 257
Exercises 4.4 259
4.5 Counting Operations in Algorithms 263
  4.5.1 Algorithms 263
  4.5.2 Pseudocode 264
  4.5.3 Sequences of Operations 266
  4.5.4 Loops 266
  4.5.5 Arrays 269
  4.5.6 Sorting 271
Exercises 4.5 273
4.6 Estimation 278
  4.6.1 Growth of Functions 278
  4.6.2 Estimation Targets 283
  4.6.3 Properties of Big-Θ 284
Exercises 4.6 285

Chapter 5 Analytical Thinking 291
5.1 Algorithms 292
  5.1.1 More Pseudocode 292
  5.1.2 Preconditions and Postconditions 294
  5.1.3 Iterative Algorithms 296
  5.1.4 Functions and Recursive Algorithms 297
Exercises 5.1 301
5.2 Three Common Types of Algorithms 305
  5.2.1 Traversal Algorithms 305
  5.2.2 Greedy Algorithms 308
  5.2.3 Divide-and-Conquer Algorithms 312
Exercises 5.2 315
5.3 Algorithm Complexity 320
  5.3.1 The Good, the Bad, and the Average 321
  5.3.2 Approximate Complexity Calculations 325
Exercises 5.3 328
5.4 Bounds on Complexity 334
  5.4.1 Algorithms as Decisions 334
  5.4.2 A Lower Bound 337
  5.4.3 Searching an Array 338
  5.4.4 Sorting 339
  5.4.5 P vs. NP 340
Exercises 5.4 341
5.5 Program Verification 345
  5.5.1 Verification versus Testing 345
  5.5.2 Verifying Recursive Algorithms 346
  5.5.3 Searching and Sorting 348
  5.5.4 Towers of Hanoi 351
Exercises 5.5 353
5.6 Loop Invariants 357
  5.6.1 Verifying Iterative Algorithms 358
  5.6.2 Searching and Sorting 361
  5.6.3 Using Invariants to Design Algorithms 364
Exercises 5.6 366

Chapter 6 Thinking Through Applications 373
6.1 Patterns in DNA 374
  6.1.1 Mutations and Phylogenetic Distance 375
  6.1.2 Phylogenetic Trees 376
  6.1.3 UPGMA 378
Exercises 6.1 382
6.2 Social Networks 384
  6.2.1 Definitions and Terminology 384
  6.2.2 Notions of Equivalence 386
  6.2.3 Hierarchical Clustering 390
  6.2.4 Signed Graphs and Balance 394
Exercises 6.2 396
6.3 Structure of Languages 398
   6.3.1 Terminology 399
   6.3.2 Finite-State Machines 400
   6.3.3 Recursion 404
   6.3.4 Further Issues in Linguistics 407
Exercises 6.3 408
6.4 Discrete-Time Population Models 410
   6.4.1 Recursive Models for Population Growth 411
   6.4.2 Fixed Points, Equilibrium, and Chaos 413
   6.4.3 Predator–Prey Systems 415
   6.4.4 The SIR Model 417
Exercises 6.4 419
6.5 Twelve-Tone Music 422
   6.5.1 Twelve-Tone Composition 423
   6.5.2 Listing All Permutations 423
   6.5.3 Transformations of Tone Rows 425
   6.5.4 Equivalence Classes and Symmetry 426
Exercises 6.5 428

Hints, Answers, and Solutions to Selected Exercises 431
1.1 Formal Logic 431
1.2 Propositional Logic 433
1.3 Predicate Logic 435
1.4 Logic in Mathematics 437
1.5 Methods of Proof 438
2.1 Graphs 440
2.2 Sets 441
2.3 Functions 443
2.4 Relations and Equivalences 445
2.5 Partial Orderings 447
2.6 Graph Theory 449
3.1 Recurrence Relations 450
3.2 Closed-Form Solutions and Induction 452
3.3 Recursive Definitions 454
3.4 Proof by Induction 455
3.5 Recursive Data Structures 458
4.1 Basic Counting Techniques 459
4.2 Selections and Arrangements 460
4.3 Counting with Functions 461
4.4 Discrete Probability 462
4.5 Counting Operations in Algorithms 463
4.6 Estimation 465
5.1 Algorithms 466
5.2 Three Common Types of Algorithms 467
5.3 Algorithm Complexity 469
5.4 Bounds on Complexity 470
5.5 Program Verification 471
5.6 Loop Invariants 472

Selected References 475

Index 479

Index of Symbols 487