Recent Topics on Mathematical Theory of Viscous Incompressible Fluid

Edited by
HIDEO KOZONO (Nagoya University)
YOSHIHIRO SHIBATA (Waseda University)
# Contents

On the Local Energy Decay Approach to Some Fluid Flow in an Exterior Domain
Wakako Dan, Takayuki Kobayashi and Yoshihiro Shibata ........................................ 1

Introduction ................................................................. 1

The Decay Property of the Stokes Semigroup in the 3-Dimensional Exterior Domain ............ 8
1.1 Main results of the section 1. ........................................ 8
1.2 Preliminaries .......................................................... 11
1.3 Resolvent expansions around the origin ............................. 15
1.4 Proof of Theorem 1.1.1 (local energy decay) ...................... 18
1.5 Proof of Theorem 1.1.2 ($L_q - L_r$ estimate) .................... 20

The Decay Property of the Stokes Semigroup in the 2-Dimensional Exterior Domain ............ 23
2.1 Main results of the section 2 ........................................ 23
2.2 Preliminaries .......................................................... 25
2.3 Asymptotic behavior of the resolvent around the origin ........... 28
2.4 Proof of Theorems 2.1.1 and 2.1.2 (local energy decay and $L_q - L_r$ estimate) .......... 33
2.5 Proof of Theorem 2.1.3 ($L_q - L_\infty$ estimate) .................. 35

The Decay Property of the Solutions to the Compressible Navier-Stokes Equation in the 3-Dimensional Exterior Domain .......................... 38
3.1 Main results for the linearized equation ........................... 38
3.2 Application to the nonlinear problem ............................... 42

The Stationary Navier-Stokes Equations in a 3D-Exterior Domain
Reinhard Farwig .................................................................... 53
1 Introduction ................................................................. 53
2 Homogeneous Sobolev Spaces ........................................... 58
3 An Existence Theorem for Weak Solutions ............................. 62
4 The Oseen Equations in $\mathbb{R}^n$ .................................... 68
5 Weighted Analysis of the Oseen Equations ............................ 73
6 On the Navier-Stokes Equations when $u_\infty \neq 0$ .................. 78
7 Basic Results on the Navier-Stokes Equations when $u_\infty = 0$ .......... 89
8 From $D$-Solutions to $PR$-Solutions when $u_\infty = 0$ ................ 98
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>Stability</td>
<td>229</td>
</tr>
<tr>
<td>0.4</td>
<td>The Cauchy problem</td>
<td>231</td>
</tr>
<tr>
<td>1</td>
<td>Results</td>
<td>232</td>
</tr>
<tr>
<td>1.1</td>
<td>Stationary problem</td>
<td>232</td>
</tr>
<tr>
<td>1.2</td>
<td>Uniqueness of weak solutions</td>
<td>234</td>
</tr>
<tr>
<td>1.3</td>
<td>Stability</td>
<td>234</td>
</tr>
<tr>
<td>1.4</td>
<td>The Cauchy problem</td>
<td>237</td>
</tr>
<tr>
<td>2</td>
<td>Outline of the proof</td>
<td>240</td>
</tr>
<tr>
<td>2.1</td>
<td>Stationary problem</td>
<td>240</td>
</tr>
<tr>
<td>2.2</td>
<td>Uniqueness criterion of the weak solution</td>
<td>244</td>
</tr>
<tr>
<td>2.3</td>
<td>Stability</td>
<td>248</td>
</tr>
<tr>
<td>2.4</td>
<td>Cauchy Problem</td>
<td>258</td>
</tr>
</tbody>
</table>