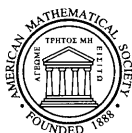


**Mathematical
Surveys
and
Monographs**
Volume 221

**A Study in Derived
Algebraic Geometry**
Volume II: Deformations,
Lie Theory
and Formal Geometry

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