The Birational Geometry of Degenerations

Robert Friedman
and David R. Morrison,
editors
CONTENTS

Preface

I. The birational geometry of degenerations: an overview
by Robert Friedman and David R. Morrison ...............1

Appendix: On the classification of surfaces with
Kodaira number zero by Robert Friedman ......26

References ...........................................29

II. Degenerations with numerically effective canonical
divisor by N. I. Shepherd-Barron .....................33

0. Introduction .........................................33
1. Degenerate cusps ....................................35
2. Elliptic double curves ..............................41
3. 0-, 1-, and 2-surfaces ............................45
4. Construction of the blowdown ......................61
5. Examples and remarks ..............................74
6. Partial compactifications of moduli spaces .......76

References ...........................................82

III. Smooth models for elliptic threefolds by Rick Miranda ......85

0. Introduction .........................................85
1. Weierstrass form ....................................86
2. The singularities of W ..............................88
3. Birational base change and preliminary reductions ....89
4. Standardization of the equation ....................91
5. Double curves ......................................93
6. The singularities of elliptic surfaces ...............93
7. The analysis of smooth points of D_0 ...............98
8. An example .........................................101
9. The blowup formula ..................................104
10. A note on normal bundles ..........................107
11. The J = 0 collisions ................................109
12. The J = 0 collisions ..............................122
13. The J = 0 collisions ................................126
14. The double point fibers ............................129
15. Some concluding remarks ...........................131

References ...........................................132
### IV. Extending polarizations on families of K3 surfaces by N. I. Shepherd-Barron

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.</td>
<td>Introduction</td>
<td>135</td>
</tr>
<tr>
<td>1.</td>
<td>Making $L$ numerically effective</td>
<td>136</td>
</tr>
<tr>
<td>2.</td>
<td>Construction of the blowdown</td>
<td>145</td>
</tr>
<tr>
<td>3.</td>
<td>Applications to birational geometry</td>
<td>158</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>160</td>
</tr>
</tbody>
</table>

#### Appendix: Linear systems on anticanonical pairs by Robert Friedman

<table>
<thead>
<tr>
<th>References for the appendix</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>162</td>
</tr>
</tbody>
</table>

### V. The minus one theorem by Rick Miranda and David R. Morrison

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Normal forms for degenerations</td>
<td>173</td>
</tr>
<tr>
<td>2.</td>
<td>Bad curves and admissible maps</td>
<td>178</td>
</tr>
<tr>
<td>3.</td>
<td>The basic algorithm</td>
<td>188</td>
</tr>
<tr>
<td>4.</td>
<td>The components of an algorithm-sequence</td>
<td>194</td>
</tr>
<tr>
<td>5.</td>
<td>Bad chains and fibrous strings</td>
<td>206</td>
</tr>
<tr>
<td>6.</td>
<td>Crucial curves</td>
<td>211</td>
</tr>
<tr>
<td>7.</td>
<td>Bad curves not in standard ruled cycles</td>
<td>223</td>
</tr>
<tr>
<td>8.</td>
<td>Standard ruled cycles with bisections</td>
<td>229</td>
</tr>
<tr>
<td>9.</td>
<td>Totally standard ruled cycles</td>
<td>239</td>
</tr>
<tr>
<td>10.</td>
<td>Special anticanonical pairs</td>
<td>244</td>
</tr>
<tr>
<td>11.</td>
<td>Special degenerations</td>
<td>252</td>
</tr>
<tr>
<td></td>
<td>Flow chart</td>
<td>257</td>
</tr>
<tr>
<td></td>
<td>Index of terminology</td>
<td>258</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>259</td>
</tr>
</tbody>
</table>

### VI. Degenerations of Kodaira surfaces by Robert Friedman and N. I. Shepherd-Barron

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td></td>
<td>261</td>
</tr>
<tr>
<td>1.</td>
<td>Calculation of the torsion</td>
<td>263</td>
</tr>
<tr>
<td>2.</td>
<td>Construction of degenerations</td>
<td>270</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>275</td>
</tr>
</tbody>
</table>

### VII. Base change, automorphisms, and stable reduction for type III K3 surfaces by Robert Friedman

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td></td>
<td>277</td>
</tr>
<tr>
<td>1.</td>
<td>Effect of base change</td>
<td>278</td>
</tr>
<tr>
<td>2.</td>
<td>Special automorphisms</td>
<td>285</td>
</tr>
<tr>
<td>3.</td>
<td>Configurations of hexagons</td>
<td>288</td>
</tr>
<tr>
<td>4.</td>
<td>Blowing down</td>
<td>292</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>297</td>
</tr>
</tbody>
</table>