Contents

Preface
Acknowledgements
List of Symbols

1 Interfacial Curvature and Contact Angle 1
1.1 Interfacial Tension 1
1.2 Young-Laplace Equation 4
1.3 The Young Equation and Contact Angle 7
  1.3.1 The Young Equation as an Energy Balance 10
  1.3.2 Interfacial Tension, Roughness and Wettability 11
  1.3.3 Capillary Rise 14
  1.3.4 Historical Interlude: Thomas Young and the Marquis de Laplace 16

2 Porous Media and Fluid Displacement 17
2.1 Pore-Space Images 17
  2.1.1 Statistical and Process-Based Pore-Space Reconstruction 22
  2.1.2 Definition of a Porous Medium, Representative Volumes, Porosity and Saturation 29
2.2 Pore-Scale Networks and Topological Description 32
  2.2.1 Transport Networks 33
  2.2.2 Network Construction 35
  2.2.3 Generalized Network Models 45
  2.2.4 Topological Descriptors of the Pore Space 50
2.3 Wettability and Displacement 56
  2.3.1 Thermodynamic Description of Displacement Processes 56
  2.3.2 Displacement Sequences 59
  2.3.3 Wettability and Wettability Change 60
  2.3.4 Surface Roughness and Contact Angle Hysteresis 65
  2.3.5 Effective Contact Angle and Curvature 69
## Contents

### 3 Primary Drainage
- 3.1 Entry Pressures and Fluid Configurations 74
  - 3.1.1 Wetting Layers 79
  - 3.1.2 Entry Pressures for Irregular Throats 82
- 3.2 Macroscopic Capillary Pressure in Drainage 85
- 3.3 Bundle of Tubes Model and the Throat Size Distribution 89
  - 3.3.1 Prediction of Capillary Pressure from Images 92
- 3.4 Invasion Percolation 95
  - 3.4.1 Scaling Relations in Invasion Percolation 99
  - 3.4.2 Displacement under Gravity and Gradient Percolation 105
  - 3.4.3 Invasion Percolation, Normal Percolation and Flow 109
- 3.5 Final Saturation and Maximum Capillary Pressure 110

### 4 Imbibition and Trapping
- 4.1 Layer Flow, Swelling and Snap-Off 116
  - 4.1.1 Roof Snap-Off during Drainage 122
- 4.2 Piston-Like Advance and Pore Filling 126
  - 4.2.1 Piston-Like Throat Filling 126
  - 4.2.2 Cooperative Pore Filling 128
  - 4.2.3 Competition between Snap-Off and Cooperative Pore Filling 132
  - 4.2.4 Frequency of Different Filling Events 135
  - 4.2.5 Dynamics of Filling 141
  - 4.2.6 Displacement as a Series of Metastable States 144
- 4.3 Displacement Patterns in Imbibition 145
  - 4.3.1 Percolation with Trapping 145
  - 4.3.2 Invasion Percolation with Trapping 147
  - 4.3.3 Frontal Advance 148
  - 4.3.4 Cluster Growth 149
  - 4.3.5 Phase Diagrams for Capillary-Controlled Displacement 149
  - 4.3.6 Infiltration or Unstable Imbibition under Gravity 157
- 4.4 Macroscopic Capillary Pressure 161
- 4.5 Interfacial Area 165
- 4.6 Capillary Trapping and Residual Saturation 168
  - 4.6.1 Direct Imaging of Trapped Clusters and Percolation Theory 168
  - 4.6.2 Effect of Initial Saturation 176

### 5 Wettability and Displacement Paths
- 5.1 Definitions and Capillary Pressure Cycles 188
Contents

5.2 Oil and Water Layers 194
  5.2.1 Pinned Water Layers and Forced Snap-Off 194
  5.2.2 Forced Water Injection and Oil Layer Formation 196
  5.2.3 Recap of Displacement Processes 200

5.3 Capillary Pressures and Wettability Indices 201
  5.3.1 Wettability Trends and Relationships between Indices 204
  5.3.2 Displacement Statistics in Mixed-Wet Systems 208

5.4 Trapping in Mixed-Wet and Oil-Wet Media 211
  5.4.1 Layer Connectivity as a Function of Initial Water Saturation 213
  5.4.2 Pore-Scale Observation of Trapping in Mixed-Wet Systems 216

6 Navier-Stokes Equations, Darcy’s Law and Multiphase Flow 219
  6.1 Navier-Stokes Equations and Conservation of Mass 219
    6.1.1 Flow in a Pipe 221
    6.1.2 The Washburn Equation 225
    6.1.3 Flow in Wetting Layers 228
    6.1.4 Reynolds Number and the Stokes Equation 233
  6.2 Darcy’s Law and Permeability 236
    6.2.1 Permeability of a Bundle of Capillary Tubes 238
    6.2.2 Typical Permeability Values 239
    6.2.3 The Leverett J Function 243
    6.2.4 Computing Flow Fields on Pore-Space Images 247
  6.3 The Multiphase Darcy Law and Relative Permeability 254
    6.3.1 Historical Interlude: Muskat, Leverett and Buckingham 254
    6.3.2 Assumptions Inherent in the Multiphase Darcy Law 255
  6.4 Capillary Number and Pore-Scale Dynamics 257
    6.4.1 Macroscopic Flow Patterns for Imbibition 257
    6.4.2 Capillary Number and the Perturbative Effect of Flow Rate 263
    6.4.3 Layer Conductance and Viscous Effects 265
    6.4.4 Correlation Lengths for Percolation-Like Displacement 270
    6.4.5 Correlation Length and Residual Saturation 273
    6.4.6 Mobilization of Trapped Ganglia 275
    6.4.7 Ganglion Dynamics, Connectivity and Flow Regimes 279
    6.4.8 Viscous and Capillary Forces as an Energy Balance 285
    6.4.9 Direct Computation of Multiphase Flow 288
  6.5 Extensions to the Multiphase Darcy Law 298
    6.5.1 Infiltration and Phase Field Models 298
Contents

6.5.2 Accounting for Non-Equilibrium Effects 300
6.5.3 Averaged Equations from Energy, Momentum and Entropy Balance 301
6.5.4 Consideration of Trapped Phases and Other Approaches 303

6.6 Flow Regimes 304
6.6.1 Dimensionless Numbers 305
6.6.2 Viscous Fingering and DLA 306
6.6.3 Summary of Regime Diagrams 309

7 Relative Permeability 315
7.1 Water-Wet Media 315
7.1.1 Primary Drainage 315
7.1.2 Waterflooding 318
7.1.3 Predictions of Relative Permeability 319
7.1.4 Relative Permeabilities for Different Rock Types 322
7.1.5 Effect of Initial Saturation 325
7.2 Effect of Wettability 326
7.2.1 Oil-Wet Media 327
7.2.2 Cross-Over Saturation and Waterflood Recovery 329
7.2.3 Mixed-Wet Media 330
7.2.4 Hysteresis in the Water Relative Permeability 333
7.2.5 Hysteresis in the Oil Relative Permeability 336
7.2.6 Features of Relative Permeability in Mixed-Wet Rocks 338
7.2.7 Guidelines for Assessing Wettability 340
7.3 Effect of Capillary Number and Viscosity Ratio 343
7.3.1 Relative Permeability in the Viscous Limit 344
7.3.2 Viscosity Ratio and Viscous Coupling 346
7.4 Empirical Models 347
7.4.1 Relative Permeability of a Bundle of Tubes 348
7.4.2 Functional Forms for Relative Permeability and Capillary Pressure 350
7.4.3 Empirical Models in Hydrology 352

8 Three-Phase Flow 354
8.1 Spreading 354
8.2 Contact Angles and the Bartell-Osterhof Equation 358
8.2.1 Wetting and Spreading in Three-Phase Flow 360
8.2.2 Why Ducks Don't Get Wet 361
8.2.3 Wettability States in Three-Phase Flow 363
8.3 Oil Layers 364
## Contents

8.3.1 Mixed-Wettability and Layer Stability 365  
8.3.2 Three Phases in Capillary/Gravity Equilibrium 366  
8.3.3 Layer Conductance and Relative Permeability 368  

8.4 Displacement Processes 373  
8.4.1 Fluid Configurations 373  
8.4.2 Configuration Changes and Layer Stability 375  
8.4.3 Multiple Displacement 375  
8.4.4 Displacement Paths 378  

8.5 Three-Phase Relative Permeability 381  
8.5.1 Pore Occupancy and Saturation Dependence 386  
8.5.2 Predictions of Three-Phase Relative Permeability 389  
8.5.3 Trapping in Three-Phase Flow 391  
8.5.4 Direct Imaging of Trapped Phases in Three-Phase Flow 397  
8.5.5 Empirical Models in Three-Phase Flow 397  

9 Solutions to Equations for Multiphase Flow 402  
9.1 Conservation Equations for Multiphase Flow 402  
9.1.1 Equations in One Dimension and the Fractional Flow 405  
9.1.2 Waterflooding and Spontaneous Imbibition 405  
9.1.3 Exemplar Relative Permeabilities and Capillary Pressures 407  
9.1.4 Boundary Conditions for One-Dimensional Flow Problems 411  

9.2 Buckley-Leverett Analysis for Two-Phase Flow 411  
9.2.1 Dimensionless Variables and Wavespeeds 413  
9.2.2 Shocks 414  
9.2.3 Constructing a Solution for Saturation 415  
9.2.4 Recovery Calculations 418  
9.2.5 Example Recovery Curves 418  

9.3 Analysis of Imbibition 422  
9.3.1 Capillary Dispersion and Fractional Flow 422  
9.3.2 Example Solutions 427  
9.3.3 Experimental Analysis of Spontaneous Imbibition 430  

9.4 Recovery, Imbibition and the Trillion-Barrel Question 432  

Appendix Exercises 437  
References 447  
Index 475  

The plate section can be found between pages 140 and 141