INTRODUCTION TO
NUMERICAL
PROGRAMMING
A Practical Guide for Scientists and Engineers
Using Python and C/C++

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