Flow, Deformation and Fracture
Lectures on Fluid Mechanics and the Mechanics of Deformable Solids for Mathematicians and Physicists

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