Contents

Preface to the Third Edition  ix
Preface   xi

CHAPTER 1 First-Order Equations  1
  1.1 The Simplest Example  1
  1.2 The Logistic Population Model  4
  1.3 Constant Harvesting and Bifurcations  7
  1.4 Periodic Harvesting and Periodic
      Solutions  10
  1.5 Computing the Poincaré Map  11
  1.6 Exploration: A Two-Parameter Family  15

CHAPTER 2 Planar Linear Systems  21
  2.1 Second-Order Differential Equations  23
  2.2 Planar Systems  24
  2.3 Preliminaries from Algebra  26
  2.4 Planar Linear Systems  29
  2.5 Eigenvalues and Eigenvectors  30
  2.6 Solving Linear Systems  33
  2.7 The Linearity Principle  36
CHAPTER 3 Phase Portraits for Planar Systems 39

3.1 Real Distinct Eigenvalues 39
3.2 Complex Eigenvalues 44
3.3 Repeated Eigenvalues 47
3.4 Changing Coordinates 49

CHAPTER 4 Classification of Planar Systems 61

4.1 The Trace–Determinant Plane 61
4.2 Dynamical Classification 64
4.3 Exploration: A 3D Parameter Space 71

CHAPTER 5 Higher-Dimensional Linear Algebra 73

5.1 Preliminaries from Linear Algebra 73
5.2 Eigenvalues and Eigenvectors 82
5.3 Complex Eigenvalues 85
5.4 Bases and Subspaces 88
5.5 Repeated Eigenvalues 93
5.6 Genericity 100

CHAPTER 6 Higher-Dimensional Linear Systems 107

6.1 Distinct Eigenvalues 107
6.2 Harmonic Oscillators 114
6.3 Repeated Eigenvalues 120
6.4 The Exponential of a Matrix 123
6.5 Nonautonomous Linear Systems 130

CHAPTER 7 Nonlinear Systems 139

7.1 Dynamical Systems 140
7.2 The Existence and Uniqueness Theorem 142
7.3 Continuous Dependence of Solutions 147
7.4 The Variational Equation 149
7.5 Exploration: Numerical Methods 153
7.6 Exploration: Numerical Methods and Chaos 156

CHAPTER 8 Equilibria in Nonlinear Systems 159

8.1 Some Illustrative Examples 159
8.2 Nonlinear Sinks and Sources 165
8.3 Saddles 168
8.4 Stability 174
8.5 Bifurcations 175
8.6 Exploration: Complex Vector Fields 182

CHAPTER 9 Global Nonlinear Techniques 187

9.1 Nullclines 187
9.2 Stability of Equilibria 192
9.3 Gradient Systems 202
9.4 Hamiltonian Systems 206
9.5 Exploration: The Pendulum with Constant Forcing 209

CHAPTER 10 Closed Orbits and Limit Sets 213

10.1 Limit Sets 213
10.2 Local Sections and Flow Boxes 216
10.3 The Poincaré Map 218
10.4 Monotone Sequences in Planar Dynamical Systems 220
10.5 The Poincaré–Bendixson Theorem 222
10.6 Applications of Poincaré–Bendixson 225
10.7 Exploration: Chemical Reactions that Oscillate 228

CHAPTER 11 Applications in Biology 233

11.1 Infectious Diseases 233
11.2 Predator–Prey Systems 237
11.3 Competitive Species 244
11.4 Exploration: Competition and Harvesting 250
11.5 Exploration: Adding Zombies to the SIR Model 251

CHAPTER 12 Applications in Circuit Theory 257

12.1 An RLC Circuit 257
12.2 The Liénard Equation 261
12.3 The van der Pol Equation 263
12.4 A Hopf Bifurcation 270
12.5 Exploration: Neurodynamics 272
CHAPTER 13 Applications in Mechanics 277

13.1 Newton's Second Law 277
13.2 Conservative Systems 280
13.3 Central Force Fields 282
13.4 The Newtonian Central Force System 285
13.5 Kepler's First Law 290
13.6 The Two-Body Problem 293
13.7 Blowing Up the Singularity 294
13.8 Exploration: Other Central Force Problems 298
13.9 Exploration: Classical Limits of Quantum Mechanical Systems 299
13.10 Exploration: Motion of a Glider 301

CHAPTER 14 The Lorenz System 305

14.1 Introduction 306
14.2 Elementary Properties of the Lorenz System 308
14.3 The Lorenz Attractor 312
14.4 A Model for the Lorenz Attractor 316
14.5 The Chaotic Attractor 321
14.6 Exploration: The Rössler Attractor 326

CHAPTER 15 Discrete Dynamical Systems 329

15.1 Introduction 329
15.2 Bifurcations 334
15.3 The Discrete Logistic Model 337
15.4 Chaos 340
15.5 Symbolic Dynamics 344
15.6 The Shift Map 349
15.7 The Cantor Middle–Thirds Set 351
15.8 Exploration: Cubic Chaos 354
15.9 Exploration: The Orbit Diagram 355

CHAPTER 16 Homoclinic Phenomena 361

16.1 The Shilnikov System 361
16.2 The Horseshoe Map 368
16.3 The Double Scroll Attractor 375
16.4 Homoclinic Bifurcations 377
16.5 Exploration: The Chua Circuit 381

CHAPTER 17 Existence and Uniqueness Revisited 385

17.1 The Existence and Uniqueness Theorem 385
17.2 Proof of Existence and Uniqueness 387
17.3 Continuous Dependence on Initial Conditions 394
17.4 Extending Solutions 397
17.5 Nonautonomous Systems 401
17.6 Differentiability of the Flow 404

Bibliography 411
Index 415