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Like the previous editions, the third edition of Dimensions is a comprehensive text on measurement designed for researchers and for use in graduate courses in psychology, education, and areas of business such as management and marketing. It is intended to consider the broad measurement problems that arise in these areas and to step aside from the specific content that often obscures these similarities. This should mean that all situations are the same. Lee Cronbach (1957) pointed out a real difference between the measurement needs of those who study differences in experimental manipulations, and those who study individual differences. This difference is noted in the pages that follow. I have also attempted to write the book so that the reader needs only a basic background in statistics.

The previous editions of this book were to widely read and accepted that there became a common denominator for more than a generation of scholars. Prof. Nunnally's death a decade ago raised the possibility that this contribution might be forgotten. I cannot, of course, know what he would have written for this edition; however, I know that I have stood with sufficient solidarity upon his shoulders. My main goal is to impress this point to the readers of this book, the force of solving measurement problems that he inspired in me. It is also with pride that I include some contributions of my own which others who have followed this path. They include Victor Hirschorn, Stanley Leiman, and Carl H. Carver.

One essential feature that I have tried to maintain is the emphasis upon variables that characterized the previous editions. Now, as here, there are many excellent references that go much further into the details of the various analyses. These are found in the many references, especially the Suggested Additional Readings at the end of each chapter. I have attempted to strike a balance between papers designed for a general audience of psychologists and graduate students, which appear in sources like Psychological Bulletin and empirical journals, versus those of a more mathematical orientation, such as Psychological and Applied Psychological Measurement.

I have also maintained the use of several examples that allow hand calculations, since many of the newer procedures do not. Consequently, I have included some procedures that many consider obsolete, specifically centroid factor analysis. Not even reader or instructor will find this useful, but I can speak from my own experience as a student on this point. Most recent developments in measurement have had it...