Quasilinear Control

Performance Analysis and Design of Feedback Systems with Nonlinear Sensors and Actuators

ShiNung Ching
Massachusetts Institute of Technology

Yongsoon Eun
Xerox Research Center Webster

Cevat Gokcek
Michigan State University

Pierre T. Kabamba
University of Michigan

Semyon M. Meerkov
University of Michigan
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