Contents

List of Figures xi
List of Tables xiii
Preface xv
Introduction xix
About the Authors xxix

1 Order and Logic 1
   1.1 Ordered Sets and Fixed-Point Theorems 1
   1.2 First-Order Predicate Logic 7
   1.3 Ordered Spaces of Valuations 12

2 The Semantics of Logic Programs 23
   2.1 Logic Programs and Their Models 23
   2.2 Supported Models 28
   2.3 Stable Models 32
   2.4 Fitting Models 37
   2.5 Perfect Models 43
   2.6 Well-Founded Models 56

3 Topology and Logic Programming 65
   3.1 Convergence Spaces and Convergence Classes 66
   3.2 The Scott Topology on Spaces of Valuations 69
   3.3 The Cantor Topology on Spaces of Valuations 76
   3.4 Operators on Spaces of Valuations Revisited 83

4 Fixed-Point Theory for Generalized Metric Spaces 87
   4.1 Distance Functions in General 88
   4.2 Metrics and Their Generalizations 91
   4.3 Generalized Ultrametrics 97
## Contents

4.4 Dislocated Metrics ........................................ 102  
4.5 Dislocated Generalized Ultrametrics .......................... 104  
4.6 Quasimetrics ............................................. 106  
4.7 A Hierarchy of Fixed-Point Theorems ........................ 112  
4.8 Relationships Between the Various Spaces ..................... 114  
4.9 Fixed-Point Theory for Multivalued Mappings ................. 125  
4.10 Partial Orders and Multivalued Mappings ..................... 127  
4.11 Metrics and Multivalued Mappings ............................ 129  
4.12 Generalized Ultrametrics and Multivalued Mappings ........... 129  
4.13 Quasimetrics and Multivalued Mappings ....................... 132  
4.14 An Alternative to Multivalued Mappings ...................... 136  

5 Supported Model Semantics ...................................... 139  
5.1 Two-Valued Supported Models ................................. 140  
5.2 Three-Valued Supported Models ............................... 151  
5.3 A Hierarchy of Logic Programs ............................... 159  
5.4 Consequence Operators and Fitting-Style Operators .......... 161  
5.5 Measurability Considerations ................................ 166  

6 Stable and Perfect Model Semantics .............................. 169  
6.1 The Fixpoint Completion ................................... 169  
6.2 Stable Model Semantics ................................... 171  
6.3 Perfect Model Semantics ................................ 175  

7 Logic Programming and Artificial Neural Networks .............. 185  
7.1 Introduction ............................................. 185  
7.2 Basics of Artificial Neural Networks ......................... 188  
7.3 The Core Method as a General Approach to Integration ....... 191  
7.4 Propositional Programs ................................... 192  
7.5 First-Order Programs .................................... 196  
7.6 Some Extensions – The Propositional Case ..................... 212  
7.7 Some Extensions – The First-Order Case ....................... 218  

8 Final Thoughts ................................................ 221  
8.1 Foundations of Programming Semantics ....................... 221  
8.2 Quantitative Domain Theory ................................ 222  
8.3 Fixed-Point Theorems for Generalized Metric Spaces .......... 223  
8.4 The Foundations of Knowledge Representation and Reasoning .... 223  
8.5 Clarifying Logic Programming Semantics ...................... 224  
8.6 Symbolic and Subsymbolic Representations ..................... 225  
8.7 Neural-Symbolic Integration ................................ 225  
8.8 Topology, Programming, and Artificial Intelligence .......... 226
# Contents

Appendix: Transfinite Induction and General Topology 229

A.1 The Principle of Transfinite Induction 229
A.2 Basic Concepts from General Topology 234
A.3 Convergence 237
A.4 Separation Properties and Compactness 238
A.5 Subspaces and Products 239
A.6 The Scott Topology 240

Bibliography 243

Index 265