Non-Equilibrium Thermodynamics of Heterogeneous Systems

S IG N E K J E L S T R U P  D I C K B E D E A U X
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