Feedback Control of Dynamic Bipedal Robot Locomotion

Eric R. Westervelt
Jessy W. Grizzle
Christine Chevallereau
Jun Ho Choi
Benjamin Morris
I Preliminaries

1 Introduction
  1.1 Why Study the Control of Bipedal Robots? ................................................. 4
  1.2 Biped Basics ................................................................. 6
    1.2.1 Terminology .......................................................... 6
    1.2.2 Dynamics ............................................................... 9
    1.2.3 Challenges Inherent to Controlling Bipedal Locomotion ......................... 11
  1.3 Overview of the Literature ......................................................... 14
    1.3.1 Polypedal Robot Locomotion ............................................. 15
    1.3.2 Bipedal Robot Locomotion ............................................... 17
    1.3.3 Control of Bipedal Locomotion ........................................... 19
  1.4 Feedback as a Mechanical Design Tool: The Notion of Virtual Constraints .......... 24
    1.4.1 Time-Invariance, or, Self-Clocking of Periodic Motions ....................... 24
    1.4.2 Virtual Constraints ................................................... 25

2 Two Test Beds for Theory ........................................................................... 29
  2.1 RABBIT ................................................................. 29
    2.1.1 Objectives of the Mechanism ........................................... 29
    2.1.2 Structure of the Mechanism ............................................. 30
    2.1.3 Lateral Stabilization ...................................................... 31
    2.1.4 Choice of Actuation ..................................................... 33
    2.1.5 Sizing the Mechanism .................................................... 33
    2.1.6 Impacts ................................................................. 35
    2.1.7 Sensors ................................................................. 35
    2.1.8 Additional Details ...................................................... 36
  2.2 ERNIE ................................................................. 37
    2.2.1 Objectives of the Mechanism ........................................... 37
    2.2.2 Enabling Continuous Walking with Limited Lab Space ......................... 38
    2.2.3 Sizing the Mechanism .................................................... 39
    2.2.4 Impacts ................................................................. 39
    2.2.5 Sensors ................................................................. 40
    2.2.6 Additional Details ...................................................... 40
II Modeling, Analysis, and Control of Robots with Passive Point Feet 43

3 Modeling of Planar Bipedal Robots with Point Feet 45
3.1 Why Point Feet? 46
3.2 Robot, Gait, and Impact Hypotheses 47
3.3 Some Remarks on Notation 52
3.4 Dynamic Model of Walking 53
3.4.1 Swing Phase Model 53
3.4.2 Impact Model 55
3.4.3 Hybrid Model of Walking 57
3.4.4 Some Facts on Angular Momentum 58
3.4.5 The MPFL-Normal Form 60
3.4.6 Example Walker Models 63
3.5 Dynamic Model of Running 71
3.5.1 Flight Phase Model 72
3.5.2 Stance Phase Model 73
3.5.3 Impact Model 74
3.5.4 Hybrid Model of Running 75
3.5.5 Some Facts on Linear and Angular Momentum 77

4 Periodic Orbits and Poincaré Return Maps 81
4.1 Autonomous Systems with Impulse Effects 82
4.1.1 Hybrid System Hypotheses 83
4.1.2 Definition of Solutions 84
4.1.3 Periodic Orbits and Stability Notions 86
4.2 Poincaré’s Method for Systems with Impulse Effects 87
4.2.1 Formal Definitions and Basic Theorems 87
4.2.2 The Poincaré Return Map as a Partial Function 90
4.3 Analyzing More General Hybrid Models 91
4.3.1 Hybrid Model with Two Continuous Phases 92
4.3.2 Basic Definitions 92
4.3.3 Existence and Stability of Periodic Orbits 94
4.4 A Low-Dimensional Stability Test Based on Finite-Time Convergence 96
4.4.1 Preliminaries 96
4.4.2 Invariance Hypotheses 96
4.4.3 The Restricted Poincaré Map 97
4.4.4 Stability Analysis Based on the Restricted Poincaré Map 97
4.5 A Low-Dimensional Stability Test Based on Timescale Separation 99
4.5.1 System Hypotheses 100
4.5.2 Stability Analysis Based on the Restricted Poincaré Map 101
6.5.2 Sample-Based Virtual Constraints and Augmentation Functions .................................................. 164

6.6 Example Controller Designs ............................................................................................................ 165
6.6.1 Designing Exponentially Stable Walking Motions without Invariance of the Impact Map ........ 165
6.6.2 Designs Based on Optimizing the HZD ................................................................................... 173
6.6.3 Designs Based on Sampled Virtual Constraints and Augmentation Functions ..................... 178

7 Systematic Design of Event-Based Feedback Controllers for Walking .............................................. 191
7.1 Overview of Key Facts ...................................................................................................................... 192
7.2 Transition Control ........................................................................................................................... 195
7.3 Event-Based PI-Control of the Average Walking Rate .................................................................... 199
7.3.1 Average Walking Rate ............................................................................................................. 199
7.3.2 Design and Analysis Based on the Hybrid Zero Dynamics ..................................................... 200
7.3.3 Design and Analysis Based on the Full-Dimensional Model .................................................. 206
7.4 Examples ......................................................................................................................................... 208
7.4.1 Choice of δα ............................................................................................................................... 208
7.4.2 Robustness to Disturbances ...................................................................................................... 210
7.4.3 Robustness to Parameter Mismatch ....................................................................................... 210
7.4.4 Robustness to Structural Mismatch ....................................................................................... 210

8 Experimental Results for Walking ....................................................................................................... 213
8.1 Implementation Issues ...................................................................................................................... 213
8.1.1 RABBIT's Implementation Issues ......................................................................................... 213
8.1.2 ERNIE's Implementation Issues ............................................................................................ 218
8.2 Control Algorithm Implementation: Imposing the Virtual Constraints ......................................... 220
8.3 Experiments ..................................................................................................................................... 225
8.3.1 Experimental Validation Using RABBIT ............................................................................... 225
8.3.2 Experimental Validation Using ERNIE ................................................................................. 241

9 Running with Point Feet ..................................................................................................................... 249
9.1 Related Work ..................................................................................................................................... 250
9.2 Qualitative Discussion of the Control Law Design ....................................................................... 251
9.2.1 Analytical Tractability through Invariance, Attractivity, and Configuration Determinism at Transitions ......................................................................................................................... 251
9.2.2 Desired Geometry of the Closed-Loop System ...................................................................... 252
9.3 Control Law Development .............................................................................................................. 254
9.3.1 Stance Phase Control ............................................................................................................... 255
9.3.2 Flight Phase Control ................................................................................................................. 256
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.3.3 Closed-Loop Hybrid Model</td>
<td>258</td>
</tr>
<tr>
<td>9.4 Existence and Stability of Periodic Orbits</td>
<td>258</td>
</tr>
<tr>
<td>9.4.1 Definition of the Poincaré Return Map</td>
<td>258</td>
</tr>
<tr>
<td>9.4.2 Analysis of the Poincaré Return Map</td>
<td>260</td>
</tr>
<tr>
<td>9.5 Example: Illustration on RABBIT</td>
<td>266</td>
</tr>
<tr>
<td>9.5.1 Stance Phase Controller Design</td>
<td>267</td>
</tr>
<tr>
<td>9.5.2 Stability of the Periodic Orbits</td>
<td>268</td>
</tr>
<tr>
<td>9.5.3 Flight Phase Controller Design</td>
<td>268</td>
</tr>
<tr>
<td>9.5.4 Simulation without Modeling Error</td>
<td>272</td>
</tr>
<tr>
<td>9.6 A Partial Robustness Evaluation</td>
<td>277</td>
</tr>
<tr>
<td>9.6.1 Compliant Contact Model</td>
<td>278</td>
</tr>
<tr>
<td>9.6.2 Simulation with Modeling Error</td>
<td>279</td>
</tr>
<tr>
<td>9.7 Additional Event-Based Control for Running</td>
<td>282</td>
</tr>
<tr>
<td>9.7.1 Deciding What to Control</td>
<td>283</td>
</tr>
<tr>
<td>9.7.2 Implementing Stride-to-Stride Updates of Landing Configuration</td>
<td>283</td>
</tr>
<tr>
<td>9.7.3 Simulation Results</td>
<td>284</td>
</tr>
<tr>
<td>9.8 Alternative Control Law Design</td>
<td>287</td>
</tr>
<tr>
<td>9.8.1 Controller Design</td>
<td>288</td>
</tr>
<tr>
<td>9.8.2 Design of Running Motions with Optimization</td>
<td>292</td>
</tr>
<tr>
<td>9.9 Experiment</td>
<td>296</td>
</tr>
<tr>
<td>9.9.1 Hardware Modifications to RABBIT</td>
<td>296</td>
</tr>
<tr>
<td>9.9.2 Result: Six Running Steps</td>
<td>296</td>
</tr>
<tr>
<td>9.9.3 Discussion</td>
<td>298</td>
</tr>
<tr>
<td>III Walking with Feet</td>
<td>299</td>
</tr>
</tbody>
</table>

### 10 Walking with Feet and Actuated Ankles

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1 Related Work</td>
<td>302</td>
</tr>
<tr>
<td>10.2 Robot Model</td>
<td>302</td>
</tr>
<tr>
<td>10.2.1 Robot and Gait Hypotheses</td>
<td>303</td>
</tr>
<tr>
<td>10.2.2 Coordinates</td>
<td>305</td>
</tr>
<tr>
<td>10.2.3 Underactuated Phase</td>
<td>305</td>
</tr>
<tr>
<td>10.2.4 Fully Actuated phase</td>
<td>306</td>
</tr>
<tr>
<td>10.2.5 Double-Support Phase</td>
<td>307</td>
</tr>
<tr>
<td>10.2.6 Foot Rotation, or Transition from Full Actuation to Underactuation</td>
<td>308</td>
</tr>
<tr>
<td>10.2.7 Overall Hybrid Model</td>
<td>309</td>
</tr>
<tr>
<td>10.2.8 Comments on the FRI Point and Angular Momentum</td>
<td>309</td>
</tr>
<tr>
<td>10.3 Creating the Hybrid Zero Dynamics</td>
<td>315</td>
</tr>
<tr>
<td>10.3.1 Control Design for the Underactuated Phase</td>
<td>315</td>
</tr>
<tr>
<td>10.3.2 Control Design for the Fully Actuated Phase</td>
<td>317</td>
</tr>
<tr>
<td>10.3.3 Transition Map from the Fully Actuated Phase to the Underactuated Phase</td>
<td>318</td>
</tr>
</tbody>
</table>
10.3.4 Transition Map from the Underactuated Phase to the Fully Actuated Phase .......................... 319
10.3.5 Hybrid Zero Dynamics ........................................... 320
10.4 Ankle Control and Stability Analysis .................................. 321
10.4.1 Analysis on the Hybrid Zero Dynamics for the Underactuated Phase .......................... 321
10.4.2 Analysis on the Hybrid Zero Dynamics for the Fully Actuated Phase with Ankle Torque Used to Change Walking Speed .......................... 322
10.4.3 Analysis on the Hybrid Zero Dynamics for the Fully Actuated Phase with Ankle Torque Used to Affect Convergence Rate .......................... 323
10.4.4 Stability of the Robot in the Full-Dimensional Model .................. 326
10.5 Designing the Virtual Constraints ..................................... 326
10.5.1 Parametrization Using Bézier polynomials .......................... 326
10.5.2 Achieving Impact Invariance of the Zero Dynamics Manifolds .......................... 328
10.5.3 Specifying the Remaining Free Parameters .......................... 330
10.6 Simulation ........................................................... 331
10.7 Special Case of a Gait without Foot Rotation .................................. 332
10.8 ZMP and Stability of an Orbit ............................................ 334

11 Directly Controlling the Foot Rotation Indicator Point .......................... 341
11.1 Introduction ........................................................... 341
11.2 Using Ankle Torque to Control FRI Position During the Fully Actuated Phase .......................... 342
11.2.1 Ability to Track a Desired Profile of the FRI Point .......................... 343
11.2.2 Analyzing the Zero Dynamics ............................................ 344
11.3 Special Case of a Gait without Foot Rotation .................................. 347
11.4 Simulations ........................................................... 348
11.4.1 Nominal Controller ....................................................... 348
11.4.2 With Modeling Errors ....................................................... 350
11.4.3 Effect of FRI Evolution on the Walking Gait .................................. 351
11.5 A Variation on FRI Position Control ............................................ 355
11.6 Simulations ........................................................... 357

A Getting Started ........................................................... 363
A.1 Graduate Student ....................................................... 363
A.2 Professional Researcher ....................................................... 368
A.2.1 Reader Already Has a Stabilizing Controller .................................. 368
A.2.2 Controller Design Must Start from Scratch .................................. 372
A.2.3 Walking with Feet ....................................................... 372
A.2.4 3D Robot ........................................................... 373
D Derivation of the Equations of Motion for Three-Dimensional Mechanisms

D.1 The Lagrangian ........................................ 457
D.2 The Kinetic Energy ...................................... 458
D.3 The Potential Energy ................................... 462
D.4 Equations of Motion .................................... 462
D.5 Invariance Properties of the Kinetic Energy ......... 464

E Single Support Equations of Motion of RABBIT ........ 465

Nomenclature .................................................. 471

End Notes ..................................................... 473

References .................................................... 479

Index .......................................................... 499

Supplemental Indices ........................................ 503