Statistical Methods for Spatio-Temporal Systems

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CHAPTER 1

Spatio-Temporal Point Processes: Methods and Applications

Peter J. Diggle

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1.1 Introduction

This chapter is concerned with the analysis of data whose basic format is 
\( (x_i, t_i) : i = 1, \ldots, n \), where each \( x_i \) denotes the location and \( t_i \) the corresponding time of occurrence of an event of interest. We shall assume that the data form a complete record of all events which occur within a pre-specified spatial region \( A \) and a prespecified time-interval, \((0, T)\). We call a data-set of this kind a spatio-temporal point pattern, and the underlying stochastic model for the data a spatio-temporal point process.

1.1.1 Motivating examples

1.1.1.1 Amacrine cells in the retina of a rabbit

One general approach to analysing spatio-temporal point process data is to extend existing methods for purely spatial data by considering the time of occurrence as a distinguishing feature, or mark, attached to each event. Before giving an example of this, we give an even simpler example of a marked spatial

![Figure 1.1](image.png) Amacrine cells in the retina of a rabbit. On and off cells are shown as open and closed circles, respectively. The rectangular region on which the cells are observed has dimension 1060 by 662 \( \mu m \).