## Contents

1 Integral Transforms  
  1.1 Brief Historical Introduction .......................... 1  
  1.2 Basic Concepts and Definitions .......................... 6  

2 Fourier Transforms and Their Applications  
  2.1 Introduction ........................................... 9  
  2.2 The Fourier Integral Formulas .......................... 10  
  2.3 Definition of the Fourier Transform and Examples .......... 12  
  2.4 Fourier Transforms of Generalized Functions ............... 17  
  2.5 Basic Properties of Fourier Transforms .................... 28  
  2.6 Poisson's Summation Formula ............................ 37  
  2.7 The Shannon Sampling Theorem ........................... 44  
  2.8 Gibbs' Phenomenon .................................... 54  
  2.9 Heisenberg's Uncertainty Principle ........................ 57  
  2.10 Applications of Fourier Transforms to Ordinary Differential Equations ........................................ 60  
  2.11 Solutions of Integral Equations ........................ 65  
  2.12 Solutions of Partial Differential Equations ............... 68  
  2.13 Fourier Cosine and Sine Transforms with Examples ........ 91  
  2.14 Properties of Fourier Cosine and Sine Transforms .......... 93  
  2.15 Applications of Fourier Cosine and Sine Transforms to Partial Differential Equations ......................... 96  
  2.16 Evaluation of Definite Integrals ........................ 100  
  2.17 Applications of Fourier Transforms in Mathematical Statistics .... 103  
  2.18 Multiple Fourier Transforms and Their Applications .......... 109  
  2.19 Exercises ........................................... 119  

3 Laplace Transforms and Their Basic Properties ................. 133  
  3.1 Introduction ........................................... 133  
  3.2 Definition of the Laplace Transform and Examples ........... 134  
  3.3 Existence Conditions for the Laplace Transform ............ 139  
  3.4 Basic Properties of Laplace Transforms .................... 140  
  3.5 The Convolution Theorem and Properties of Convolution ... 145  
  3.6 Differentiation and Integration of Laplace Transforms ........ 151  
  3.7 The Inverse Laplace Transform and Examples ............... 154  
  3.8 Tauberian Theorems and Watson's Lemma ..................... 168
4 Applications of Laplace Transforms

4.1 Introduction ......................................................... 181
4.2 Solutions of Ordinary Differential Equations ...................... 182
4.3 Partial Differential Equations, Initial and Boundary Value Problems ....................................................... 207
4.4 Solutions of Integral Equations ..................................... 222
4.5 Solutions of Boundary Value Problems .............................. 225
4.6 Evaluation of Definite Integrals ..................................... 228
4.7 Solutions of Difference and Differential-Difference Equations .......................................................... 230
4.8 Applications of the Joint Laplace and Fourier Transform ....... 237
4.9 Summation of Infinite Series ......................................... 248
4.10 Transfer Function and Impulse Response Function of a Linear System ....................................................... 251
4.11 Exercises .......................................................... 256

5 Fractional Calculus and Its Applications

5.1 Introduction .......................................................... 269
5.2 Historical Comments .................................................. 270
5.3 Fractional Derivatives and Integrals ................................ 272
5.4 Applications of Fractional Calculus ................................ 279
5.5 Exercises .......................................................... 282

6 Applications of Integral Transforms to Fractional Differential and Integral Equations

6.1 Introduction .......................................................... 283
6.2 Laplace Transforms of Fractional Integrals and Fractional Derivatives ....................................................... 284
6.3 Fractional Ordinary Differential Equations ......................... 287
6.4 Fractional Integral Equations ......................................... 290
6.5 Initial Value Problems for Fractional Differential Equations .......................................................... 295
6.6 Green’s Functions of Fractional Differential Equations ........... 298
6.7 Fractional Partial Differential Equations ............................ 299
6.8 Exercises .......................................................... 312

7 Hankel Transforms and Their Applications

7.1 Introduction .......................................................... 315
7.2 The Hankel Transform and Examples ................................ 316
7.3 Operational Properties of the Hankel Transform ................. 319
7.4 Applications of Hankel Transforms to Partial Differential Equations ....................................................... 322
7.5 Exercises .......................................................... 331
8 Mellin Transforms and Their Applications 339
  8.1 Introduction ........................................ 339
  8.2 Definition of the Mellin Transform and Examples .... 340
  8.3 Basic Operational Properties of Mellin Transforms .... 343
  8.4 Applications of Mellin Transforms .................. 349
  8.5 Mellin Transforms of the Weyl Fractional Integral and the Weyl Fractional Derivative .............. 353
  8.6 Application of Mellin Transforms to Summation of Series .... 358
  8.7 Generalized Mellin Transforms ...................... 361
  8.8 Exercises ........................................... 365

9 Hilbert and Stieltjes Transforms 371
  9.1 Introduction ........................................ 371
  9.2 Definition of the Hilbert Transform and Examples .... 372
  9.3 Basic Properties of Hilbert Transforms ............... 375
  9.4 Hilbert Transforms in the Complex Plane .............. 378
  9.5 Applications of Hilbert Transforms .................. 380
  9.6 Asymptotic Expansions of One-Sided Hilbert Transforms .... 388
  9.7 Definition of the Stieltjes Transform and Examples .... 391
  9.8 Basic Operational Properties of Stieltjes Transforms .... 394
  9.9 Inversion Theorems for Stieltjes Transforms .......... 396
  9.10 Applications of Stieltjes Transforms ................ 399
  9.11 The Generalized Stieltjes Transform ................ 401
  9.12 Basic Properties of the Generalized Stieltjes Transform .... 403
  9.13 Exercises ........................................... 404

10 Finite Fourier Sine and Cosine Transforms 407
  10.1 Introduction ....................................... 407
  10.2 Definitions of the Finite Fourier Sine and Cosine Transforms and Examples ............... 408
  10.3 Basic Properties of Finite Fourier Sine and Cosine Transforms ....................... 410
  10.4 Applications of Finite Fourier Sine and Cosine Transforms .............. 416
  10.5 Multiple Finite Fourier Transforms and Their Applications .............. 422
  10.6 Exercises ........................................... 425

11 Finite Laplace Transforms 429
  11.1 Introduction ....................................... 429
  11.2 Definition of the Finite Laplace Transform and Examples .............. 430
  11.3 Basic Operational Properties of the Finite Laplace Transform .............. 436
  11.4 Applications of Finite Laplace Transforms ................ 439
  11.5 Tauberian Theorems ................................ 443
  11.6 Exercises ........................................... 443
### 12 Z Transforms

- **12.1 Introduction** .................................................. 445
- **12.2 Dynamic Linear Systems and Impulse Response** .................. 445
- **12.3 Definition of the Z Transform and Examples** .................. 449
- **12.4 Basic Operational Properties of Z Transforms** ............... 453
- **12.5 The Inverse Z Transform and Examples** ....................... 459
- **12.6 Applications of Z Transforms to Finite Difference Equations** 463
- **12.7 Summation of Infinite Series** ............................... 466
- **12.8 Exercises** ....................................................... 469

### 13 Finite Hankel Transforms

- **13.1 Introduction** .................................................. 473
- **13.2 Definition of the Finite Hankel Transform and Examples** .... 473
- **13.3 Basic Operational Properties** ................................ 476
- **13.4 Applications of Finite Hankel Transforms** .................... 476
- **13.5 Exercises** ....................................................... 481

### 14 Legendre Transforms

- **14.1 Introduction** .................................................. 485
- **14.2 Definition of the Legendre Transform and Examples** .......... 486
- **14.3 Basic Operational Properties of Legendre Transforms** ....... 489
- **14.4 Applications of Legendre Transforms to Boundary Value Problems** 497
- **14.5 Exercises** ....................................................... 498

### 15 Jacobi and Gegenbauer Transforms

- **15.1 Introduction** .................................................. 501
- **15.2 Definition of the Jacobi Transform and Examples** ............. 501
- **15.3 Basic Operational Properties** ................................ 504
- **15.4 Applications of Jacobi Transforms to the Generalized Heat Conduction Problem** 505
- **15.5 The Gegenbauer Transform and Its Basic Operational Properties** 507
- **15.6 Application of the Gegenbauer Transform** .................... 510

### 16 Laguerre Transforms

- **16.1 Introduction** .................................................. 511
- **16.2 Definition of the Laguerre Transform and Examples** .......... 511
- **16.3 Basic Operational Properties** ................................ 516
- **16.4 Applications of Laguerre Transforms** ........................ 520
- **16.5 Exercises** ....................................................... 523