QUANTUM WELLS, WIRES AND DOTS
Theoretical and Computational Physics of Semiconductor Nanostructures

Second Edition

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