Contents

Preface to the Third Edition xv
Preface to the Second Edition xvii
Preface to the First Edition xix

1 Introduction 1

1.1. Multivariate Statistical Analysis, 1
1.2. The Multivariate Normal Distribution, 3

2 The Multivariate Normal Distribution 6

2.1. Introduction, 6
2.2. Notions of Multivariate Distributions, 7
2.3. The Multivariate Normal Distribution, 13
2.4. The Distribution of Linear Combinations of Normally Distributed Variates; Independence of Variates; Marginal Distributions, 23
2.5. Conditional Distributions and Multiple Correlation Coefficient, 33
2.6. The Characteristic Function; Moments, 41
2.7. Elliptically Contoured Distributions, 47
Problems, 56

3 Estimation of the Mean Vector and the Covariance Matrix 66

3.1. Introduction, 66
3.2. The Maximum Likelihood Estimators of the Mean Vector and the Covariance Matrix, 67
3.3. The Distribution of the Sample Mean Vector; Inference Concerning the Mean When the Covariance Matrix Is Known, 74
3.4. Theoretical Properties of Estimators of the Mean Vector, 83
3.5. Improved Estimation of the Mean, 91
3.6. Elliptically Contoured Distributions, 101
Problems, 108

4 The Distributions and Uses of Sample Correlation Coefficients 115

4.1. Introduction, 115
4.2. Correlation Coefficient of a Bivariate Sample, 116
4.3. Partial Correlation Coefficients; Conditional Distributions, 136
4.4. The Multiple Correlation Coefficient, 144
4.5. Elliptically Contoured Distributions, 158
Problems, 163

5 The Generalized $T^2$-Statistic 170

5.1. Introduction, 170
5.2. Derivation of the Generalized $T^2$-Statistic and Its Distribution, 171
5.3. Uses of the $T^2$-Statistic, 177
5.4. The Distribution of $T^2$ under Alternative Hypotheses; The Power Function, 185
5.5. The Two-Sample Problem with Unequal Covariance Matrices, 187
5.6. Some Optimal Properties of the $T^2$-Test, 190
5.7. Elliptically Contoured Distributions, 199
Problems, 201

6 Classification of Observations 207

6.1. The Problem of Classification, 207
6.2. Standards of Good Classification, 208
6.3. Procedures of Classification into One of Two Populations with Known Probability Distributions, 211
CONTENTS

6.4. Classification into One of Two Known Multivariate Normal Populations, 215
6.5. Classification into One of Two Multivariate Normal Populations When the Parameters Are Estimated, 219
6.6. Probabilities of Misclassification, 227
6.7. Classification into One of Several Populations, 233
6.8. Classification into One of Several Multivariate Normal Populations, 237
6.9. An Example of Classification into One of Several Multivariate Normal Populations, 240
6.10. Classification into One of Two Known Multivariate Normal Populations with Unequal Covariance Matrices, 242
Problems, 248

7 The Distribution of the Sample Covariance Matrix and the Sample Generalized Variance 251

7.1. Introduction, 251
7.2. The Wishart Distribution, 252
7.3. Some Properties of the Wishart Distribution, 258
7.4. Cochran's Theorem, 262
7.5. The Generalized Variance, 264
7.6. Distribution of the Set of Correlation Coefficients When the Population Covariance Matrix Is Diagonal, 270
7.7. The Inverted Wishart Distribution and Bayes Estimation of the Covariance Matrix, 272
7.8. Improved Estimation of the Covariance Matrix, 276
7.9. Elliptically Contoured Distributions, 282
Problems, 285

8 Testing the General Linear Hypothesis; Multivariate Analysis of Variance 291

8.1. Introduction, 291
8.2. Estimators of Parameters in Multivariate Linear Regression, 292
8.3. Likelihood Ratio Criteria for Testing Linear Hypotheses about Regression Coefficients, 298
8.4. The Distribution of the Likelihood Ratio Criterion When the Hypothesis Is True, 304
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5</td>
<td>An Asymptotic Expansion of the Distribution of the Likelihood RatioCriterion</td>
<td>316</td>
</tr>
<tr>
<td>8.6</td>
<td>Other Criteria for Testing the Linear Hypothesis</td>
<td>326</td>
</tr>
<tr>
<td>8.7</td>
<td>Tests of Hypotheses about Matrices of Regression Coefficients and Confidence Regions</td>
<td>337</td>
</tr>
<tr>
<td>8.8</td>
<td>Testing Equality of Means of Several Normal Distributions with Common Covariance Matrix</td>
<td>342</td>
</tr>
<tr>
<td>8.9</td>
<td>Multivariate Analysis of Variance</td>
<td>346</td>
</tr>
<tr>
<td>8.10</td>
<td>Some Optimal Properties of Tests</td>
<td>353</td>
</tr>
<tr>
<td>8.11</td>
<td>Elliptically Contoured Distributions</td>
<td>370</td>
</tr>
<tr>
<td></td>
<td>Problems</td>
<td>374</td>
</tr>
<tr>
<td>9</td>
<td>Testing Independence of Sets of Variates</td>
<td>381</td>
</tr>
<tr>
<td>9.1</td>
<td>Introduction</td>
<td>381</td>
</tr>
<tr>
<td>9.2</td>
<td>The Likelihood Ratio Criterion for Testing Independence of Sets of Variates</td>
<td>381</td>
</tr>
<tr>
<td>9.3</td>
<td>The Distribution of the Likelihood Ratio Criterion When the Null Hypothesis Is True</td>
<td>386</td>
</tr>
<tr>
<td>9.4</td>
<td>An Asymptotic Expansion of the Distribution of the Likelihood Ratio Criterion</td>
<td>390</td>
</tr>
<tr>
<td>9.5</td>
<td>Other Criteria</td>
<td>391</td>
</tr>
<tr>
<td>9.6</td>
<td>Step-Down Procedures</td>
<td>393</td>
</tr>
<tr>
<td>9.7</td>
<td>An Example</td>
<td>396</td>
</tr>
<tr>
<td>9.8</td>
<td>The Case of Two Sets of Variates</td>
<td>397</td>
</tr>
<tr>
<td>9.9</td>
<td>Admissibility of the Likelihood Ratio Test</td>
<td>401</td>
</tr>
<tr>
<td>9.10</td>
<td>Monotonicity of Power Functions of Tests of Independence of Sets</td>
<td>402</td>
</tr>
<tr>
<td>9.11</td>
<td>Elliptically Contoured Distributions</td>
<td>404</td>
</tr>
<tr>
<td></td>
<td>Problems</td>
<td>408</td>
</tr>
<tr>
<td>10</td>
<td>Testing Hypotheses of Equality of Covariance Matrices and Equality of Mean Vectors and Covariance Matrices</td>
<td>411</td>
</tr>
<tr>
<td>10.1</td>
<td>Introduction</td>
<td>411</td>
</tr>
<tr>
<td>10.2</td>
<td>Criteria for Testing Equality of Several Covariance Matrices</td>
<td>412</td>
</tr>
<tr>
<td>10.3</td>
<td>Criteria for Testing That Several Normal Distributions Are Identical</td>
<td>415</td>
</tr>
<tr>
<td>10.4</td>
<td>Distributions of the Criteria</td>
<td>417</td>
</tr>
</tbody>
</table>
10.5. Asymptotic Expansions of the Distributions of the Criteria, 424
10.6. The Case of Two Populations, 427
10.7. Testing the Hypothesis That a Covariance Matrix Is Proportional to a Given Matrix; The Sphericity Test, 431
10.8. Testing the Hypothesis That a Covariance Matrix Is Equal to a Given Matrix, 438
10.9. Testing the Hypothesis That a Mean Vector and a Covariance Matrix Are Equal to a Given Vector and Matrix, 444
10.10. Admissibility of Tests, 446
10.11. Elliptically Contoured Distributions, 449
Problems, 454

11 Principal Components

11.1. Introduction, 459
11.2. Definition of Principal Components in the Population, 460
11.3. Maximum Likelihood Estimators of the Principal Components and Their Variances, 467
11.4. Computation of the Maximum Likelihood Estimates of the Principal Components, 469
11.5. An Example, 471
11.6. Statistical Inference, 473
11.7. Testing Hypotheses about the Characteristic Roots of a Covariance Matrix, 478
11.8. Elliptically Contoured Distributions, 482
Problems, 483

12 Canonical Correlations and Canonical Variables

12.1. Introduction, 487
12.2. Canonical Correlations and Variates in the Population, 488
12.3. Estimation of Canonical Correlations and Variates, 498
12.4. Statistical Inference, 503
12.5. An Example, 505
12.6. Linearly Related Expected Values, 508
12.7. Reduced Rank Regression, 514
12.8. Simultaneous Equations Models, 515
Problems, 526

13 The Distributions of Characteristic Roots and Vectors 528

13.1. Introduction, 528
13.2. The Case of Two Wishart Matrices, 529
13.3. The Case of One Nonsingular Wishart Matrix, 538
13.4. Canonical Correlations, 543
13.5. Asymptotic Distributions in the Case of One Wishart Matrix, 545
13.6. Asymptotic Distributions in the Case of Two Wishart Matrices, 549
13.7. Asymptotic Distribution in a Regression Model, 555
13.8. Elliptically Contoured Distributions, 563
Problems, 567

14 Factor Analysis 569

14.1. Introduction, 569
14.2. The Model, 570
14.3. Maximum Likelihood Estimators for Random Orthogonal Factors, 576
14.4. Estimation for Fixed Factors, 586
14.5. Factor Interpretation and Transformation, 587
14.6. Estimation for Identification by Specified Zeros, 590
14.7. Estimation of Factor Scores, 591
Problems, 593

15 Patterns of Dependence; Graphical Models 595

15.1. Introduction, 595
15.2. Undirected Graphs, 596
15.3. Directed Graphs, 604
15.4. Chain Graphs, 610
15.5. Statistical Inference, 613

Appendix A Matrix Theory 624

A.1. Definition of a Matrix and Operations on Matrices, 624
A.2. Characteristic Roots and Vectors, 631
CONTENTS

A.3. Partitioned Vectors and Matrices, 635
A.4. Some Miscellaneous Results, 639
A.5. Gram–Schmidt Orthogonalization and the Solution of
Linear Equations, 647

Appendix B Tables

B.1. Wilks' Likelihood Criterion: Factors $C(p, m, M)$ to
Adjust to $\chi^2_{p,m}$, where $M = n - p + 1$, 651
B.2. Tables of Significance Points for the Lawley-Hotelling
Trace Test, 657
B.3. Tables of Significance Points for the
Bartlett–Nanda–Pillai Trace Test, 673
B.4. Tables of Significance Points for the Roy Maximum Root
Test, 677
B.5. Significance Points for the Modified Likelihood Ratio
Test of Equality of Covariance Matrices Based on Equal
Sample Sizes, 681
B.6. Correction Factors for Significance Points for the
Sphericity Test, 683
B.7. Significance Points for the Modified Likelihood Ratio
Test $\Sigma = \Sigma_0$, 685

References

Index