Limit Theorems for Markov Chains and Stochastic Properties of Dynamical Systems by Quasi-Compactness
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Generalization to the non-ergodic case, by L. Hervé

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