CONTENTS

1 GREEN FUNCTIONS IN THE BCS THEORY

1 Introduction
   1.1 Superconducting variables
      1.1.1 Ginzburg–Landau theory
      1.1.2 Example: Vortices in type II superconductors
      1.1.3 Bogoliubov–de Gennes equations
      1.1.4 Quasiclassical approximation
   1.2 Nonstationary phenomena
      1.2.1 Time-dependent Ginzburg–Landau theory
      1.2.2 Microscopic argumentation
      1.2.3 Boltzmann kinetic equation
   1.3 Outline of the contents

2 Green functions
   2.1 Second quantization
      2.1.1 Schrödinger and Heisenberg operators
   2.2 Imaginary-time Green function
      2.2.1 Definitions
      2.2.2 Example: Free particles
      2.2.3 The Wick theorem
   2.3 The real-time Green functions
      2.3.1 Definitions
      2.3.2 Analytical properties

3 The BCS model
   3.1 BCS theory and Gor’kov equations
      3.1.1 Magnetic field
      3.1.2 Frequency and momentum representation
      3.1.3 Order parameter of a d-wave superconductor
   3.2 Derivation of the Bogoliubov–de Gennes equations
   3.3 Thermodynamic potential
   3.4 Example: Homogeneous state
      3.4.1 Green functions
      3.4.2 Gap equation for an s-wave superconductor
   3.5 Perturbation theory
      3.5.1 Diagram technique
      3.5.2 Electric current
## CONTENTS

4 Superconducting alloys
   4.1 Averaging over impurity positions
      4.1.1 Magnetic impurities
   4.2 Homogeneous state of an s-wave superconductor

II QUASICLASSICAL METHOD

5 General principles of the quasiclassical approximation
   5.1 Quasiclassical Green functions
   5.2 Density, current, and order parameter
   5.3 Homogeneous state
   5.4 Real-frequency representation
      5.4.1 Example: Homogeneous state
   5.5 Eilenberger equations
      5.5.1 Self-energy
      5.5.2 Normalization
   5.6 Dirty limit. Usadel equations
   5.7 Boundary conditions
      5.7.1 Diffusive surface

6 Quasiclassical methods in stationary problems
   6.1 s-wave superconductors with impurities
      6.1.1 Small currents in a uniform state
      6.1.2 Ginzburg–Landau theory
      6.1.3 The upper critical field in a dirty alloy
   6.2 Gapless s-wave superconductivity
      6.2.1 Critical temperature
      6.2.2 Gap in the energy spectrum
   6.3 Aspects of d-wave superconductivity
      6.3.1 Impurities and d-wave superconductivity
      6.3.2 Impurity-induced gapless excitations
      6.3.3 The Ginzburg–Landau equations
   6.4 Bound states in vortex cores
      6.4.1 Superconductors with s-wave pairing
      6.4.2 d-wave superconductors

7 Quasiclassical method for layered superconductors
   7.1 Quasiclassical Green functions
   7.2 Eilenberger equations for layered systems
   7.3 Lawrence–Doniach model
      7.3.1 Order parameter
      7.3.2 Free energy and the supercurrent
      7.3.3 Microscopic derivation of the supercurrent
   7.4 Applications of the Lawrence–Doniach model
      7.4.1 Upper critical field
      7.4.2 Intrinsic pinning
CONTENTS

III NONEQUILIBRIUM SUPERCONDUCTIVITY

8 Nonstationary theory
  8.1 The method of analytical continuation 143
    8.1.1 Clean superconductors 145
    8.1.2 Impurities 149
    8.1.3 Order parameter, current, and particle density 152
  8.2 The phonon model 152
    8.2.1 Self-energy 152
    8.2.2 Order parameter 157
  8.3 Particle–particle collisions 159
  8.4 Transport-like equations and the conservation laws 161
  8.5 The Keldysh diagram technique 163
    8.5.1 Definitions of the Keldysh functions 163
    8.5.2 Dyson equation 167
    8.5.3 Keldysh functions in the BCS theory 168

9 Quasiclassical method for nonstationary phenomena 170
  9.1 Eliashberg equations 170
    9.1.1 Self-energies 172
    9.1.2 Order parameter, current, and particle density 174
    9.1.3 Normalization of the quasiclassical functions 174
  9.2 Generalized distribution function 175
  9.3 s-wave superconductors with a short mean free path 177
  9.4 Stimulated superconductivity 181

10 Kinetic equations 186
  10.1 Gauge-invariant Green functions 186
    10.1.1 Equations of motion for the invariant functions 188
  10.2 Quasiclassical kinetic equations 192
    10.2.1 Superconductors in electromagnetic fields 194
    10.2.2 Discussion 197
  10.3 Observables in the gauge-invariant representation 199
    10.3.1 The electron density and charge neutrality 201
  10.4 Collision integrals 203
    10.4.1 Impurities 204
    10.4.2 Electron–phonon collision integral 205
    10.4.3 Electron–electron collision integral 207
  10.5 Kinetic equations for dirty s-wave superconductors 209
    10.5.1 Small gradients without magnetic impurities 210
    10.5.2 Heat conduction 211

11 The time-dependent Ginzburg–Landau theory 213
  11.1 Gapless superconductors with magnetic impurities 213
  11.2 Generalized TDGL equations 215
  11.3 TDGL theory for d-wave superconductors 221
  11.4 d.c. electric field in superconductors. Charge imbalance 226
IV VORTEX DYNAMICS

12 Time-dependent Ginzburg–Landau analysis 231
   12.1 Introduction 231
   12.2 Energy balance 233
   12.3 Moving vortex 234
   12.4 Force balance 236
   12.5 Flux flow 238
       12.5.1 Single vortex: Low fields 238
       12.5.2 Dense lattice: High fields 240
       12.5.3 Direction of the vortex motion 242
   12.6 Anisotropic superconductors 243
       12.6.1 Low fields 245
       12.6.2 High fields 246
   12.7 Flux flow in layered superconductors 246
       12.7.1 Motion of pancake vortices 247
       12.7.2 Intrinsic pinning 247
   12.8 Flux flow within a generalized TDGL theory 248
       12.8.1 Dirty superconductors 248
       12.8.2 d-wave superconductors 251
       12.8.3 Discussion: Flux flow conductivity 252
   12.9 Flux flow Hall effect 253
       12.9.1 Modified TDGL equations 254
       12.9.2 Hall effect: Low fields 255
       12.9.3 High fields 256
       12.9.4 Discussion: Hall effect 256

13 Vortex dynamics in dirty superconductors 259
   13.1 Microscopic derivation of the force on moving vortices 259
       13.1.1 Variation of the thermodynamic potential 259
       13.1.2 Force on vortices 260
   13.2 Diffusion controlled flux flow 263
       13.2.1 Discussion 269

14 Vortex dynamics in clean superconductors 271
   14.1 Introduction 271
       14.1.1 Boltzmann kinetic equation approach 272
       14.1.2 Forces in s-wave superconductors 274
   14.2 Spectral representation for the Green functions 277
   14.3 Useful identities 279
   14.4 Distribution function 282
       14.4.1 Localized excitations 282
       14.4.2 Delocalized excitations 287
   14.5 Flux flow conductivity 290
   14.6 Discussion 292
       14.6.1 Conductivity: Low temperatures 292