Contents

Preface xi

Chapter 1  Fundamental Concepts 1

1.1 What Is a Graph? 1
   The Definition, 1
   Graphs as Models, 3
   Matrices and Isomorphism, 6
   Decomposition and Special Graphs, 11
   Exercises, 14

1.2 Paths, Cycles, and Trails 19
   Connection in Graphs, 20
   Bipartite Graphs, 24
   Eulerian Circuits, 26
   Exercises, 31

1.3 Vertex Degrees and Counting 34
   Counting and Bijections, 35
   Extremal Problems, 38
   Graphic Sequences, 44
   Exercises, 47

1.4 Directed Graphs 53
   Definitions and Examples, 53
   Vertex Degrees, 58
   Eulerian Digraphs, 60
   Orientations and Tournaments, 61
   Exercises, 63
# Chapter 2  Trees and Distance

## 2.1 Basic Properties
- Properties of Trees, 68
- Distance in Trees and Graphs, 70
- Disjoint Spanning Trees (optional), 73
- Exercises, 75

## 2.2 Spanning Trees and Enumeration
- Enumeration of Trees, 81
- Spanning Trees in Graphs, 83
- Decomposition and Graceful Labelings, 87
- Branchings and Eulerian Digraphs (optional), 89
- Exercises, 92

## 2.3 Optimization and Trees
- Minimum Spanning Tree, 95
- Shortest Paths, 97
- Trees in Computer Science (optional), 100
- Exercises, 103

# Chapter 3  Matchings and Factors

## 3.1 Matchings and Covers
- Maximum Matchings, 108
- Hall's Matching Condition, 110
- Min-Max Theorems, 112
- Independent Sets and Covers, 113
- Dominating Sets (optional), 116
- Exercises, 118

## 3.2 Algorithms and Applications
- Maximum Bipartite Matching, 123
- Weighted Bipartite Matching, 125
- Stable Matchings (optional), 130
- Faster Bipartite Matching (optional), 132
- Exercises, 134

## 3.3 Matchings in General Graphs
- Tutte's 1-factor Theorem, 136
- \( f \)-factors of Graphs (optional), 140
- Edmonds' Blossom Algorithm (optional), 142
- Exercises, 145
Chapter 6  Planar Graphs  

6.1  Embeddings and Euler’s Formula  
Drawings in the Plane, 233 
Dual Graphs, 236 
Euler’s Formula, 241 255  
Exercises, 243 

6.2  Characterization of Planar Graphs  
Preparation for Kuratowski’s Theorem, 247 
Convex Embeddings, 248 
Planarity Testing (optional), 252  
Exercises, 255 

6.3  Parameters of Planarity  
Coloring of Planar Graphs, 257 
Crossing Number, 261 
Surfaces of Higher Genus (optional), 266  
Exercises, 269 

Chapter 7  Edges and Cycles  

7.1  Line Graphs and Edge-coloring  
Edge-colorings, 274 
Characterization of Line Graphs (optional), 279  
Exercises, 282 

7.2  Hamiltonian Cycles  
Necessary Conditions, 287 
Sufficient Conditions, 288 
Cycles in Directed Graphs (optional), 293 
Exercises, 294 

7.3  Planarity, Coloring, and Cycles  
Tait’s Theorem, 300 
Grinberg’s Theorem, 302 
Snarks (optional), 304 
Flows and Cycle Covers (optional), 307 
Exercises, 314
Chapter 8  Additional Topics (optional) 319

8.1  Perfect Graphs 319
  The Perfect Graph Theorem, 320
  Chordal Graphs Revisited, 323
  Other Classes of Perfect Graphs, 328
  Imperfect Graphs, 334
  The Strong Perfect Graph Conjecture, 340
  Exercises, 344

8.2  Matroids 349
  Hereditary Systems and Examples, 349
  Properties of Matroids, 354
  The Span Function, 358
  The Dual of a Matroid, 360
  Matroid Minors and Planar Graphs, 363
  Matroid Intersection, 366
  Matroid Union, 369
  Exercises, 372

8.3  Ramsey Theory 378
  The Pigeonhole Principle Revisited, 378
  Ramsey’s Theorem, 380
  Ramsey Numbers, 383
  Graph Ramsey Theory, 386
  Sperner’s Lemma and Bandwidth, 388
  Exercises, 392

8.4  More Extremal Problems 396
  Encodings of Graphs, 397
  Branchings and Gossip, 404
  List Coloring and Choosability, 408
  Partitions Using Paths and Cycles, 413
  Circumference, 416
  Exercises, 422

8.5  Random Graphs 425
  Existence and Expectation, 426
  Properties of Almost All Graphs, 430
  Threshold Functions, 432
  Evolution and Graph Parameters, 436
  Connectivity, Cliques, and Coloring, 439
  Martingales, 442
  Exercises, 448