Binding and Scattering in Two-Dimensional Systems

Applications to Quantum Wires, Waveguides and Photonic Crystals
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