INTRODUCTION TO CONTROL OF OSCILLATIONS AND CHAOS

A. L. Fradkov
A. Yu. Pogromskiy
Russian Academy of Sciences
Contents

Preface v

Notations and definitions 1

Chapter 1 Introduction 5
1.1 What is control? ........................................... 5
  1.1.1 Models of the controlled plants .................. 6
  1.1.2 Control goals ..................................... 8
  1.1.3 "Naïve" control .................................. 10
  1.1.4 Feedback ....................................... 12
  1.1.5 Uncertainty .................................... 13
  1.1.6 Nonlinearity ................................... 15
1.2 What is chaos? ..................................... 18
1.3 What use is it? .................................... 21
  1.3.1 Mechanics and mechanical engineering .......... 21
  1.3.2 Electrical engineering and telecommunications .. 22
  1.3.3 Chemistry and chemical engineering .......... 24
  1.3.4 Biology, biochemistry and medicine .......... 26
  1.3.5 Economics and finance ....................... 27

Chapter 2 The mathematics of nonlinear control 29
2.1 Mathematical models of controlled systems .......... 29
2.2 Stability and boundedness ................. 37
  2.2.1 State-space stability ....................... 37
  2.2.2 Stability theorems and Lyapunov functions .. 43
  2.2.3 Absolute stability ....................... 53
## Contents

2.3 Feedback linearization and normal forms .................................. 61  
2.4 Feedback stabilization and passivity ...................................... 67  
  2.4.1 Introductory comments ............................................. 67  
  2.4.2 Passivity and dissipativity ....................................... 69  
  2.4.3 Passification as a control design problem ....................... 78  
  2.4.4 Input-to-state stability .......................................... 83  
2.5 Speed gradient algorithms .............................................. 87  
  2.5.1 Goal-oriented formulation of the control problem ............ 87  
  2.5.2 Design of Speed Gradient Algorithms ............................. 90  
  2.5.3 Properties of the speed gradient algorithms .................... 94  
  2.5.4 Identifying properties of SG algorithms ........................ 110  
2.6 Robustness of speed gradient algorithms with respect to disturbances ........................................ 112  
2.7 Gradient control of discrete-time systems ............................ 116  

Chapter 3  The mathematics of oscillations and chaos 121  
3.1 What is oscillation? ...................................................... 121  
  3.1.1 General concepts ................................................. 121  
  3.1.2 Oscillations in dynamical systems ................................ 125  
3.2 Stability of oscillations ................................................ 132  
  3.2.1 Convergence and synchronization ................................ 132  
  3.2.2 Lyapunov stability, Lyapunov exponents, Bol exponents ...... 139  
  3.2.3 Computation of the Bol exponents ................................ 144  
  3.2.4 Orbital stability .................................................. 145  
3.3 Poincaré maps ............................................................. 149  
  3.3.1 Definition and properties of the Poincaré map ................. 149  
  3.3.2 Controlled Poincaré maps ....................................... 151  
  3.3.3 Controlled closing lemma ........................................ 156  
3.4 What is chaos? (continued) ........................................... 158  

Chapter 4  Methods of nonlinear and adaptive control of oscillations 163  
4.1 Adaptive control problem statement .................................. 163  
4.2 Direct and identification approaches to adaptive control design 169  
4.3 Adaptive systems with reference models ............................ 173  
  4.3.1 Problem statement ................................................. 173  
  4.3.2 State feedback ..................................................... 174  
  4.3.3 Output feedback .................................................. 182
4.4 Controlled synchronization of dynamical systems 186
4.5 Decomposition based synchronization 189
4.6 Passivity based synchronization 193
  4.6.1 Semipassivity and $\mathcal{L}$-dissipativity 193
  4.6.2 Synchronization of two linearly coupled systems 196
  4.6.3 Synchronization of several systems with multiple interconnections 203
  4.6.4 Adaptive synchronization 207
  4.6.5 Adaptive synchronization of uncertain semipassive systems 210
  4.6.6 Adaptive synchronization of hyper-minimum-phase systems 215
4.7 Adaptive suppression of forced oscillations 218
4.8 Control of cascaded systems. Relaxation of the matching condition 232
  4.8.1 Integrator backstepping 232
  4.8.2 Adaptive control of unmatched systems 238
4.9 Speed Gradient control of Hamiltonian systems 247
  4.9.1 Control of energy 247
  4.9.2 The swinging (small control) property 252
  4.9.3 Control of first integrals 253
  4.9.4 Control of generalized Hamiltonian systems 257
4.10 Discrete adaptive control via linearization of Poincaré map 260
  4.10.1 Background and motivation 260
  4.10.2 Linearization of the controlled Poincaré map 261
4.11 Control of bifurcations 269

Chapter 5 Control of oscillatory and chaotic systems 273
5.1 Control of pendulums 273
  5.1.1 Swinging a simple pendulum 273
  5.1.2 Pendulum with a controlled suspension point 280
5.2 Stabilization of the equilibrium point of the thermal convection loop model 286
5.3 Adaptive synchronization of two forced Duffing's systems 295
5.4 Adaptive synchronization of Chua's circuits 304
5.5 Gradient control of the Hénon system 309
  5.5.1 Stabilizing the unstable equilibrium of the Hénon system 310
Contents

5.5.2 Synchronizing two identical Hénon systems ........... 312
5.5.3 Adaptive model reference control of the Hénon system . 313
5.6 Control of periodic and chaotic oscillations in the brussellator model .................................................. 314

Chapter 6 Applications 323
6.1 How to tow a car out of a ditch ................................. 323
6.2 Synchronization of generators based on tunnel diodes .......... 327
6.3 Stabilization of swings in power systems ...................... 333
6.4 Adaptive control of the thin film growth from a multicomponent gas ......................................................... 342
6.5 Control of oscillatory behavior of populations .................. 346
6.6 Control of a nonlinear business-cycle model ................... 351

Chapter 7 Conclusions: What is the message of the book? 359

Exercises 363
Bibliography 367
Index 389