Contents

Chapter 1
Complex Numbers

1.1 Introduction ........................................... 1
1.2 More Properties of Complex Numbers .............. 8
1.3 Complex Numbers and the Argand Plane ......... 12
1.4 Integer and Fractional Powers of a Complex Number .... 26
1.5 Loci, Points, Sets, and Regions in the Complex Plane .... 36

Chapter 2
The Complex Function and Its Derivative

2.1 Introduction ........................................... 45
2.2 Limits and Continuity .................................. 50
2.3 The Complex Derivative ............................. 58
2.4 The Derivative and Analyticity .................... 64
2.5 Harmonic Functions ................................... 73
2.6 Some Physical Applications of Harmonic Functions .... 80

Chapter 3
The Basic Transcendental Functions

3.1 The Exponential Function ........................... 93
3.2 Trigonometric Functions ............................ 98
Chapter 4
Integration in the Complex Plane

4.1 Introduction to Line Integration
4.2 Complex Line Integration
4.3 Contour Integration and Green's Theorem
4.4 Path Independence and Indefinite Integrals
4.5 The Cauchy Integral Formula and Its Extension
4.6 Some Applications of the Cauchy Integral Formula
4.7 Introduction to Dirichlet Problems—The Poisson Integral Formula for the Circle and Half Plane
Appendix: Green's Theorem in the Plane

Chapter 5
Infinite Series Involving a Complex Variable

5.1 Introduction and Review of Real Series
5.2 Convergence of Complex Series
5.3 Uniform Convergence of Series
5.4 Power Series and Taylor Series
5.5 Techniques for Obtaining Taylor Series Expansions
5.6 Laurent Series
5.7 Some Properties of Analytic Functions Related to Taylor Series
Appendix A: Sequences, Fractals, and the Mandelbrot Set
Appendix B: The z Transformation

Chapter 6
Residues and Their Use in Integration

6.1 Definition of the Residue
6.2 Isolated Singularities
6.3 Finding the Residue 327
6.4 Evaluation of Real Integrals with Residue Calculus, I 335
6.5 Evaluation of Integrals, II 339
6.6 Evaluation of Integrals, III 348
6.7 Integrals Involving Indented Contours 360
6.8 Contour Integrations Involving Branch Points and Branch Cuts 367
6.9 Integration Around Infinity as a Tool for Evaluating
Definite Integrals 375
6.10 Residue Calculus Applied to Fourier Transforms 389
Appendix: The Use of Residues to Sum Certain Series 401

Chapter 7

Laplace Transforms and Stability of Systems

7.1 Introduction to and Inversion of Laplace Transforms 415
7.2 Stability—An Introduction 444
7.3 Principle of the Argument 455
7.4 The Nyquist Stability Criterion 464
7.5 Laplace Transforms and Stability with Generalized Functions 472

Chapter 8

Conformal Mapping and Some of Its Applications

8.1 Introduction 491
8.2 The Conformal Property 493
8.3 One-to-One Mappings and Mappings of Regions 503
8.4 The Bilinear Transformation 513
8.5 Conformal Mapping and Boundary Value Problems 532
8.6 More on Boundary Value Problems—Streamlines as Boundaries 553
8.7 Boundary Value Problems with Sources 564
8.8 The Schwarz-Christoffel Transformation 584
Appendix: The Stream Function and Capacitance 600

Selected Answers 605

Index 617