Introduction to Nonlinear Physics

With 264 Figures
# Contents

**Preface**

<table>
<thead>
<tr>
<th>1 Introduction</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lui Lam</td>
<td></td>
</tr>
<tr>
<td>1.1 A Quiet Revolution</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Nonlinearity</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Nonlinear Science</td>
<td>4</td>
</tr>
<tr>
<td>1.3.1 Fractals</td>
<td>4</td>
</tr>
<tr>
<td>1.3.2 Chaos</td>
<td>5</td>
</tr>
<tr>
<td>1.3.3 Pattern Formation</td>
<td>6</td>
</tr>
<tr>
<td>1.3.4 Solitons</td>
<td>6</td>
</tr>
<tr>
<td>1.3.5 Cellular Automata</td>
<td>7</td>
</tr>
<tr>
<td>1.3.6 Complex Systems</td>
<td>8</td>
</tr>
<tr>
<td>1.4 Remarks</td>
<td>9</td>
</tr>
<tr>
<td>References</td>
<td>10</td>
</tr>
</tbody>
</table>

## Part I Fractals and Multifractals

<table>
<thead>
<tr>
<th>2 Fractals and Diffusive Growth</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thomas C. Halsey</td>
<td></td>
</tr>
<tr>
<td>2.1 Percolation</td>
<td>16</td>
</tr>
<tr>
<td>2.2 Diffusion-Limited Aggregation</td>
<td>17</td>
</tr>
<tr>
<td>2.3 Electrostatic Analogy</td>
<td>20</td>
</tr>
<tr>
<td>2.4 Physical Applications of DLA</td>
<td>22</td>
</tr>
<tr>
<td>2.4.1 Electrodeposition with Secondary Current Distribution</td>
<td>24</td>
</tr>
<tr>
<td>2.4.2 Diffusive Electrodeposition</td>
<td>27</td>
</tr>
<tr>
<td>Problems</td>
<td>28</td>
</tr>
<tr>
<td>References</td>
<td>28</td>
</tr>
</tbody>
</table>
5.7.5 Regular Attractors 75
5.7.6 Review of Stability 82
5.8 Bifurcations 82
5.9 Chaos 89
5.9.1 Binary Shift Map 90
5.9.2 Chaos in Flows 92
5.9.3 The Rössler Attractor 94
5.9.4 The Lorenz Attractor 97
5.9.5 Stable and Unstable Manifolds 98
5.10 Homoclinic Tangle 100
5.10.1 Chaos in Higher Dimensions 101
5.10.2 Bifurcations Between Chaotic Attractors 102
Problems 103
References 105

6 Probability, Random Processes, and the Statistical Description of Dynamics 106
Stephen G. Eubank and J. Doyne Farmer

6.1 Nondeterminism in Dynamics 106
6.2 Measure and Probability 107
6.2.1 Estimating a Density Function from Data 110
6.3 Nondeterministic Dynamics 113
6.4 Averaging 115
6.4.1 Stationarity 115
6.4.2 Time Averages and Ensemble Averages 116
6.4.3 Mixing 120
6.5 Characterization of Distributions 122
6.5.1 Moments 122
6.5.2 Entropy and Information 133
6.6 Fractals, Dimension, and the Uncertainty Exponent 138
6.6.1 Pointwise Dimension 139
6.6.2 Information Dimension 140
6.6.3 Fractal Dimension 140
6.6.4 Generalized Dimensions 141
6.6.5 Estimating Dimension from Data 142
6.6.6 Embedding Dimension 144
6.6.7 Fat Fractals 144
6.6.8 Lyapunov Dimension 145
6.6.9 Metric Entropy 146
6.6.10 Pesin's Identity 148
6.7 Dimensions, Lyapunov Exponents, and Metric Entropy in the Presence of Noise 148
Problems 149
References 150
Part IV Solitons

10 Integrable Systems

Lui Lam

10.1 Introduction

10.2 Origin and History of Solitons

10.3 Integrability and Conservation Laws

10.4 Soliton Equations and their Solutions

10.4.1 Korteweg–de Vries Equation

10.4.2 Nonlinear Schrödinger Equation

10.4.3 Sine–Gordon Equation

10.4.4 Kadomtsev–Petviashvili Equation

10.5 Methods of Solution

10.5.1 Inverse Scattering Method

10.5.2 Bäcklund Transformation

10.5.3 Hirota Method

10.5.4 Numerical Method

10.6 Physical Soliton Systems

10.6.1 Shallow Water Waves

10.6.2 Dislocations in Crystals

10.6.3 Self-Focusing of Light

10.7 Conclusions

Problems

References

11 Nonintegrable Systems

Lui Lam

11.1 Introduction

11.2 Nonintegrable Soliton Equations with Hamiltonian Structures

11.2.1 The $\theta^4$ Equation

11.2.2 Double Sine–Gordon Equation

11.3 Nonlinear Evolution Equations

11.3.1 Fisher Equation

11.3.2 The Damped $\theta^4$ Equation

11.3.3 The Damped Driven Sine–Gordon Equation

11.4 A Method of Constructing Soliton Equations

11.5 Formation of Solitons

11.6 Perturbations
11.7 Soliton Statistical Mechanics
  11.7.1 The $\theta^4$ System 248
  11.7.2 The Sine–Gordon System 251
11.8 Solitons in Condensed Matter
  11.8.1 Liquid Crystals 252
  11.8.2 Polyacetylene 261
  11.8.3 Optical Fibers 264
  11.8.4 Magnetic Systems 266
11.9 Conclusions 266
Problems 267
References 268

Part V Special Topics

12 Cellular Automata and Discrete Physics 275
   David E. Hiebeler and Robert Tatar

12.1 Introduction 275
   12.1.1 A Well-Known Example: Life 277
   12.1.2 Cellular Automata 278
   12.1.3 The Information Mechanics Group 279
12.2 Physical Modeling 280
   12.2.1 CA Quasiparticles 280
   12.2.2 Physical Properties from CA Simulations 281
   12.2.3 Diffusion 282
   12.2.4 Sound Waves 285
   12.2.5 Optics 287
   12.2.6 Chemical Reactions 289
12.3 Hardware 290
12.4 Current Sources of Literature 291
12.5 An Outstanding Problem in CA Simulations 291
Problems 292
References 294

13 Visualization Techniques for Cellular Dynamata 296
   Ralph H. Abraham

13.1 Historical Introduction 296
13.2 Cellular Dynamata 297
   13.2.1 Dynamical Schemes 297
   13.2.2 Complex Dynamical Systems 297
   13.2.3 CD Definitions 297
   13.2.4 CD States 299
   13.2.5 CD Simulation 299
   13.2.6 CD Visualization 299
13.3 An Example of Zeeman’s Method 300
   13.3.1 Zeeman’s Heart Model: Standard Cell 300
   13.3.2 Zeeman’s Heart Model: Physical Space 300
13.3.3 Zeeman's Heart Model: Beating 300
13.4 The Graph Method 300
  13.4.1 The Biased Logistic Scheme 301
  13.4.2 The Logistic/Diffusion Lattice 301
  13.4.3 The Global State Graph 302
13.5 The Isochron Coloring Method 305
  13.5.1 Isochrons of a Periodic Attractor 305
  13.5.2 Coloring Strategies 305
13.6 Conclusions 306
References 306

14 From Laminar Flow to Turbulence 308
Geoffrey K. Vallis

14.1 Preamble and Basic Ideas 308
  14.1.1 What Is Turbulence? 309
14.2 From Laminar Flow to Nonlinear Equilibration 311
  14.2.1 A Linear Analysis: The Kelvin–Helmholz Instability 312
  14.2.2 A Weakly Nonlinear Analysis: Landau's Equation 314
14.3 From Nonlinear Equilibration to Weak Turbulence 321
  14.3.1 The Quasi-Periodic Sequence 322
  14.3.2 The Period Doubling Sequence 324
  14.3.3 The Intermittent Sequence 335
  14.3.4 Fluid Relevance and Experimental Evidence 337
14.4 Strong Turbulence 341
  14.4.1 Scaling Arguments for Inertial Ranges 341
  14.4.2 Predictability of Strong Turbulence 348
  14.4.3 Renormalizing the Diffusivity 352
14.5 Remarks 355
References 357

15 Active Walks: Pattern Formation, Self-Organization, and Complex Systems 359
Lui Lam

15.1 Introduction 359
15.2 Basic Concepts 360
15.3 Continuum Description 361
15.4 Computer Models 363
  15.4.1 A Single Walker 363
  15.4.2 Branching 366
  15.4.3 Multiwalkers and Updating Rules 366
  15.4.4 Track Patterns 368
15.5 Three Applications 371
  15.5.1 Dielectric Breakdown in a Thin Layer of Liquid 371
  15.5.2 Ion Transport in Glasses 375
  15.5.3 Ant Trails in Food Collection 376
15.6 Intrinsic Abnormal Growth 378