## Contents

C. Generating Functions ........................................... 16  
D. Cumulant Generating Function ................................ 17  
E. Characteristic Functions ....................................... 17  

### Multivariate Distributions

- A. Discrete Case .............................................. 17  
- B. Continuous Case ........................................... 17  

### Moments

- Marginal and Conditional Distributions ....................... 18  

### Probability Distributions

- A. Discrete Case .............................................. 19  
  1. Discrete Uniform Distribution ............................ 19  
  2. Binomial Distribution .................................... 19  
  3. Geometric Distribution ................................... 20  
  4. Multinomial Distribution ................................ 20  
  5. Poisson Distribution ...................................... 20  
  6. Hypergeometric Distribution .............................. 20  
  7. Negative Binomial Distribution ......................... 21  
- B. Continuous ................................................... 21  
  1. Uniform Distribution ..................................... 21  
  2. Normal Distribution ...................................... 21  
  3. Gamma Distribution ....................................... 22  
  4. Exponential Distribution .................................. 22  
  5. Beta Distribution ........................................ 22  

### Sampling Distributions

- 1. Chi-Square Distribution .................................... 23  
  Reproductive Property of Chi-Square Distribution ......... 23  
- 2. Snedecor's F-Distribution .................................. 23  
- 3. Student's t-Distribution ................................... 24  

### Summary of Significance Tests

- Testing for the Value of a Specified Parameter ............ 25  
- Comparison of Two Populations ............................... 26  

### Summary of Confidence Intervals


### Analysis of Variance (ANOVA) Tables

- One-Way Classification ....................................... 28  
- Two-Way Classification With One Observation Per Cell .... 29  
- Nested Classifications With Unequal Samples ............... 30  
- Nested Classifications With Equal Samples ................ 31  
- Fixed Model Two-Factor Factorial Experiment in a One-Way Classification Design .... 32  
- Fixed Model Three-Factor Factorial Experiment in a Completely Randomized Design .... 33  
- Latin Square .................................................... 35  
- Graeco-Latin Square .......................................... 36  
- Youden Square .................................................. 36  
- Balanced Incomplete Block (BIB) ............................. 37  

### General Linear Model

- 1. Notation ..................................................... 37  
- 2. General Linear Model ...................................... 38  
  2.1 The Simple Regression Model ............................ 38  
  2.2 Multiple Regression Model ............................... 39  
  2.3 One-Way Classification Analysis of Variance .......... 39  
  2.4 Two-Way Classification (Two Factors Factorial) ...... 40  
  2.5 Analysis of Covariance ................................... 41  
- 3. Summary of Rules for Matrix Operations ................. 41  
  3.1 Expectation .............................................. 41  
  3.2 Partitioning of Determinants ............................ 41  
  3.3 Inverse of a Partitioned Matrix ........................ 42  
  3.4 Characteristic Roots ..................................... 42  
  3.5 Differentiation .......................................... 42  
  3.6 Some Additional Definitions and Rules ................. 43  
- 4. Principle of Minimizing Quadratic Forms and Gauss-Markov Theorem .... 44  
  4.1 Some Remarks on Multivariate Distributions .......... 44  
  4.2 The Principle of Least Squares .......................... 45  
  4.3 Minimum Variance Unbiased Estimates .................. 45
5. General Linear Hypothesis of Full Rank ........................................ 46
   5.1 Notation ................................................................................. 46
   5.2 Simple Linear Regression ..................................................... 47
   5.3 Analysis of Variance, One-Way Classification ......................... 47
   5.4 Multiple Linear Regression .................................................. 48
   5.5 Randomized Blocks ............................................................... 49
   5.6 Quadratic Form Due to Hypothesis ......................................... 50
   5.7 Sum of Squares Due to Error .................................................. 50
   5.8 Summary ............................................................................... 51
   5.9 Computational Procedure for Testing a Hypothesis .................. 51
   5.10 Regression Significance Test ................................................ 52
   5.11 Alternate Form of the Distribution ........................................ 52

6. General Linear Model of Less Than Full Rank ............................. 52
   6.1 Estimable Function and Estimability ..................................... 53
   6.2 General Linear Hypothesis Model of Less Than Full Rank ......... 55
   6.3 Constraints and Conditions .................................................... 56

Simplified Computations for Multiple Regression .......................... 58
Algebraic Procedure ................................................................. 58
Algebraic Procedure for the Forward Solution of the Abbreviated Doolittle Method .................................................. 60
Tests of Significance ....................................................................... 61
Backward Solution of the Abbreviated Doolittle Method ................. 62

Plans for Design of Experiments ................................................. 64
Selected Latin Squares .................................................................. 64
Graeco-Latin Squares ................................................................. 65
Index to Plans of Factorial Experiments Confounded in Randomized Incomplete Blocks.............................. 66
Confounded Designs for Other Factorial Experiments ....................... 67
Index to Plans for 2^n Factorials in Fractional Replication ............... 73
Index to Plans, Incomplete Block Designs ................................... 84
Index to Plans, Incomplete Latin Squares ................................... 93

Main Effect and Interactions in 2^1, 2^2, 2^3 and 2^4 Factorial Designs ............................................................. 106

Finite Differences ....................................................................... 110
Function Build-up from Differences .............................................. 113
Interpolation ............................................................................... 113
   Newton's Forward Formula ...................................................... 113
   Newton's Backward Formula .................................................. 114
   Gauss' Forward Formula ......................................................... 114
   Gauss' Backward Formula ....................................................... 114
   Stirling's Formula .................................................................. 114
   Steffenson's Formula .............................................................. 114
   Bessel's Formula ................................................................... 114
   Everett's Formula ................................................................. 114
   Bessel's Formula (unmodified) ................................................. 116
   Everett's Formula (unmodified) .............................................. 116
   Generalized Throwback .......................................................... 118
   Symmetric Formulae for Interpolation to Halves ......................... 118
   Interpolation Techniques Which Do Not Require the Function to be Tabulated for Equal Interval of the Argument ............................................................. 119
      a) Lagrangian Polynomials .................................................... 119
      b) Divided Differences .......................................................... 119
      c) Adjusted Divided Differences ............................................ 120
      d) Iterative Linear Interpolation ............................................ 122
      e) Gauss' Trigonometric Interpolation Formula ....................... 122
      f) Reciprocal Differences ..................................................... 123
Inverse Interpolation .................................................................... 124

Part II—NORMAL DISTRIBUTION

II.1 The Normal Probability Function and Related Functions ............ 125
II.2 Tolerance Factors for Normal Distributions ............................... 135
II.3 Factors for Computing Probable Errors .................................. 140
II.4 Probability of Occurrence of Deviations .................................. 143
II.5 Operating Characteristic (OC) Curves for a Test on the Mean of a Normal Distribution With Known Standard Deviation .................. 144
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.6 Bivariate Normal Probabilities</td>
<td>147</td>
</tr>
<tr>
<td>11.7 Circular Normal Probabilities</td>
<td>151</td>
</tr>
<tr>
<td>11.8 Circular Error Probabilities</td>
<td>154</td>
</tr>
<tr>
<td>11.9 Charts of Upper 1%, 2.5%, and 5% Points of the Distribution of the Largest Characteristic Root</td>
<td>157</td>
</tr>
<tr>
<td>11.10 Probit Analysis</td>
<td>170</td>
</tr>
</tbody>
</table>

### Part III—BINOMIAL, POISSON, HYPERGEOMETRIC, AND NEGATIVE BINOMIAL DISTRIBUTIONS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>III.1 Individual Terms, Binomial Distribution</td>
<td>182</td>
</tr>
<tr>
<td>III.2 Cumulative Terms, Binomial Distribution</td>
<td>194</td>
</tr>
<tr>
<td>III.3 Individual Terms, Poisson Distribution</td>
<td>206</td>
</tr>
<tr>
<td>III.4 Cumulative Terms, Poisson Distribution</td>
<td>212</td>
</tr>
<tr>
<td>III.5 Confidence Limits for Proportions</td>
<td>219</td>
</tr>
<tr>
<td>III.6 Confidence Limits for the Expected Value of a Poisson Distribution</td>
<td>238</td>
</tr>
<tr>
<td>III.7 Various Functions of $p$ and $q = 1 - p$</td>
<td>240</td>
</tr>
<tr>
<td>III.8 Hypergeometric Distribution</td>
<td>245</td>
</tr>
<tr>
<td>III.9 Negative Binomial Distribution</td>
<td>250</td>
</tr>
<tr>
<td>III.10 Percentage Points of the Beta Distribution</td>
<td>251</td>
</tr>
<tr>
<td>III.11 Tests of Significance in $2 \times 2$ Contingency Tables</td>
<td>266</td>
</tr>
</tbody>
</table>

### Part IV—STUDENT’S $t$-DISTRIBUTION

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV.1 Percentage Points, Student’s $t$-Distribution</td>
<td>282</td>
</tr>
<tr>
<td>IV.2 Power Function of the $t$-Test</td>
<td>284</td>
</tr>
<tr>
<td>IV.3 Number of Observations for $t$-Test of Mean</td>
<td>286</td>
</tr>
<tr>
<td>IV.4 Number of Observations for $t$-Test of Difference Between Two Means</td>
<td>288</td>
</tr>
<tr>
<td>IV.5 Operating Characteristic (OC) Curves for a Test on the Mean of a Normal Distribution With Unknown Standard Deviation</td>
<td>290</td>
</tr>
</tbody>
</table>

### Part V—CHI-SQUARE DISTRIBUTION

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>V.1 Percentage Points, Chi-Square Distribution</td>
<td>293</td>
</tr>
<tr>
<td>V.2 Percentage Points, Chi-Square Over Degrees of Freedom Distribution</td>
<td>295</td>
</tr>
<tr>
<td>V.3 Number of Observations for the Comparison of a Population Variance With a Standard Value Using the Chi-Square Test</td>
<td>299</td>
</tr>
<tr>
<td>V.4 Operating Characteristic (OC) Curves for a Test on the Standard Deviation of a Normal Distribution</td>
<td>300</td>
</tr>
</tbody>
</table>

### Part VI—F-DISTRIBUTION

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>VI.1 Percentage Points, F-Distribution</td>
<td>304</td>
</tr>
<tr>
<td>VI.2 Power Functions of the Analysis-of-Variance Tests</td>
<td>311</td>
</tr>
<tr>
<td>VI.3 Number of Observations Required for the Comparison of Two Population Variances Using the F-Test</td>
<td>320</td>
</tr>
<tr>
<td>VI.4 Operating Characteristic (OC) Curves for a Test on the Standard Deviation of Two Normal Distributions</td>
<td>322</td>
</tr>
<tr>
<td>VI.5 Cochran’s Test for the Homogeneity of Variances</td>
<td>325</td>
</tr>
<tr>
<td>VI.6 Percentage Points of the Maximum F-Ratio</td>
<td>328</td>
</tr>
</tbody>
</table>

### Part VII—ORDER STATISTICS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>VII.1 Expected Values of Order Statistics From a Standard Normal Population</td>
<td>330</td>
</tr>
<tr>
<td>VII.2 Variances and Covariances of Order Statistics</td>
<td>332</td>
</tr>
<tr>
<td>VII.3 Confidence Intervals for Medians</td>
<td>338</td>
</tr>
<tr>
<td>VII.4 Critical Values for Testing Outliers</td>
<td>339</td>
</tr>
<tr>
<td>VII.5 Percentile Estimates in Large Samples</td>
<td>346</td>
</tr>
<tr>
<td>VII.6 Simple Estimates in Small Samples</td>
<td>348</td>
</tr>
</tbody>
</table>

### Part VIII—RANGE AND STUDENTIZED RANGE

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIII.1 Probability Integral of the Range</td>
<td>351</td>
</tr>
<tr>
<td>VIII.2 Percentage Points, Distribution of the Range</td>
<td>360</td>
</tr>
<tr>
<td>VIII.3 Percentage Points, Studentized Range</td>
<td>361</td>
</tr>
<tr>
<td>VIII.4 Critical Values for Duncan’s New Multiple Range Test</td>
<td>368</td>
</tr>
<tr>
<td>VIII.5 Substitute t-Ratios</td>
<td>379</td>
</tr>
</tbody>
</table>
Contents

VIII.6 Substitute F-Ratio .................................................. 382
VIII.7 Analysis of Variance Based on Range ................................ 385
VIII.8 Confidence Intervals for $\sigma$ Based on Mean Range .......... 387

Part IX—CORRELATION COEFFICIENT
IX.1 Percentage Points, Distribution of the Correlation Coefficient when $\rho = 0$ .............. 389
IX.2 Confidence Limits for the Population Correlation Coefficient ............... 391
IX.3 The Transformation $z = \tanh^{-1} r$ for the Correlation Coefficient ............... 394

Part X—NON-PARAMETRIC STATISTICS
X.1 Critical Values for the Sign Test ..................................... 397
X.2 Critical Values of T in the Wilcoxon Matched-Pairs Signed-Ranks Test .................. 399
X.3 Probabilities for the Wilcoxon (Mann-Whitney) Two-Sample Statistic ................. 401
X.4 Critical Values of U in the Wilcoxon (Mann-Whitney) Two-Sample Statistic ......... 405
X.5 Critical Values for the Wilcoxon Rank Sum Test ........................... 409
X.6 Distribution of the Total Number-of-Runs Test ................................ 414
X.7 Critical Values for the Kolmogorov-Smirnov One-Sample Statistic .................. 425
X.8 Critical Values for the Kolmogorov-Smirnov Two-Sample Statistic .................. 427
X.9 Kruskal-Wallis One-Way Analysis of Variance by Ranks ......................... 430
X.10 Critical Values for a Sum of Ranks Procedure for Relative Spread in Unpaired Samples ................................................................. 433
X.11 Significant Values for a Rank-Sum Test for Dispersion ......................... 442
X.12 Critical Values of Spearman's Rank Correlation Coefficient ............. 445
X.13 Critical Values of Kendall's Rank Correlation Coefficient .................. 449

Part XI—QUALITY CONTROL
XI.1 Factors for Computing Control Limits ................................ 451
XI.2 Percentage Points of the Distribution of the Mean Deviation .................. 455
XI.3 Cumulative Sum Control Charts (CSCC) .................................. 456
   A. CSCC for the Mean ............................................. 456
   B. CSCC for Sample Ranges ....................................... 458
   C. CSCC for Sample Variances ..................................... 459
   D. CSCC for Number of Defectives, $np$, or Fraction Defective $p$ ............... 461
   E. CSCC for Number of Defects, $c$ .................................. 463
   F. Summary of CSCC Limits ....................................... 464

Part XII—MISCELLANEOUS STATISTICAL TABLES
XII.1 Number of Permutations ............................................ 466
XII.2 Number of Combinations ............................................ 467
XII.3 Logarithms of the Binomial Coefficients .................................. 472
XII.4 Random Units .......................................................... 479
XII.5 Random Normal Numbers, $\mu = 0, \sigma = 1$ ............................ 484
XII.6 Random Normal Numbers, $\mu = 2, \sigma = 1$ ............................ 494
XII.7 Random Normal Numbers, $\mu = 0, \sigma = 2$ ............................ 499
XII.8 Orthogonal Polynomials ............................................. 504
XII.9 Percentage Points of Pearson Curves ................................... 518

Part XIII—MISCELLANEOUS MATHEMATICAL TABLES
XIII.1 Miscellaneous Constants ........................................... 527
XIII.2 Numerical Constants ................................................ 528
XIII.3 Radians to Degrees, Minutes, and Seconds ............................. 529
XIII.4 Natural Functions for Angles in Radians ................................ 530
XIII.5 Squares, Cubes, and Roots ......................................... 532
XIII.6 Exponential Functions ............................................... 549
XIII.7 Six-Place Logarithms ................................................ 557
XIII.8 Natural or Naperian Logarithms ...................................... 579
XIII.9 Factorials and Their Logarithms ..................................... 587
XIII.10 Reciprocals of Factorials and Their Logarithms ......................... 589
XIII.11 Powers of Numbers ................................................ 590
XIII.12 Sums of Powers of Integers ....................................... 592
XIII.13 Integrals ............................................................. 594
XIII.14 Gamma Function ..................................................... 635

Index ................................................................. 637