# Contents

Preface .................................................. xi

1. Introduction ........................................... 1
   1.1 Stability of Linear Systems ...................... 5
   1.2 Variance of Linear Stochastic Systems .......... 8
   1.3 Quadratic Performance Measure ................. 9
   1.4 Book Organization ............................... 11
   1.5 References ...................................... 15

2. Continuous Algebraic Lyapunov Equation ............ 21
   2.1 Explicit Solutions ............................... 22
      2.1.1 Expansion Methods ......................... 22
      2.1.2 Skew-Symmetric Matrix Approach .......... 30
      2.1.3 Special Cases ................................ 31
   2.2 Solution Bounds ................................ 39
      2.2.1 Eigenvalue Bounds ........................... 40
      2.2.2 Trace Bounds ................................ 56
      2.2.3 Determinant Bounds ......................... 59
   2.3 Numerical Solutions .............................. 62
      2.3.1 Bartels and Stewart Algorithm ............ 66
   2.4 Summary ......................................... 70
   2.5 References ...................................... 71

3. Discrete Algebraic Lyapunov Equation .............. 79
   3.1 Explicit Solutions ............................... 80
      3.1.1 Bilinear Transformation .................... 80
      3.1.2 Jordan Form Technique ...................... 83
      3.1.3 Skew-Symmetric Matrix Approach .......... 84
   3.2 Bounds of Solution's Attributes ............... 85
      3.2.1 Eigenvalue Bounds ........................... 85
      3.2.2 Trace Bounds ................................ 90
      3.2.3 Determinant Bounds ......................... 94
   3.3 Numerical Solutions .............................. 99
   3.4 Summary ......................................... 101
   3.5 References ...................................... 102
## 7. Iterative Methods and Parallel Algorithms

- **7.1 Smith's Algorithm** ............................................. 170
- **7.2 ADI Iterative Method** ........................................ 172
- **7.3 SOR Iterative Method** ....................................... 174
- **7.4 Parallel Algorithms** .......................................... 175
- **7.5 Parallel Algorithms for Coupled Lyapunov Equations** .... 178
  - **7.5.1 Continuous Coupled Algebraic Lyapunov Equations** .... 178
  - **7.5.2 Discrete Coupled Algebraic Lyapunov Equations** ...... 181
- **7.6 Comments** .................................................... 184
- **7.7 References** .................................................. 185

## 8. Lyapunov Iterations

- **8.1 Kleinman's Algorithm for Riccati Equation** ............... 190
- **8.2 Lyapunov Iterations for Jump Parameter Linear Systems** . 195
- **8.3 Lyapunov Iterations for Nash Differential Games** ......... 208
- **8.4 Lyapunov Iterations for Output Feedback Control** ....... 215
  - **8.4.1 Case Study: Fluid Catalytic Cracker** ............... 217
- **8.5 Comments** .................................................... 218
- **8.6 References** .................................................. 218

## 9. Concluding Remarks

- **9.1 Sylvester Equations** ......................................... 223
- **9.2 Related Topics** ............................................... 227
- **9.3 Applications** ................................................ 228
  - **9.3.1 Case Study: Magnetic Tape Control System** .......... 231
  - **9.3.2 Case Study: Aircraft under Wind Disturbances** ..... 232
- **9.4 Comments** .................................................... 233
- **9.5 References** .................................................. 234

### Appendix
- **Matrix Inequalities** ............................................. 243

### Index
- .................................................. 251