CONTENTS

Introduction 1

PART I Mathematical Structures and Their Theories

1. Relational Systems 9
2. Boolean Algebras 13
3. Subsystems and Homomorphisms 19
4. Operations on Relational Systems 25
5. Terms and Formulas 30
6. Theories and Models 47
7. Substitution of Terms 55
8. Theorems and Proofs 62
9. Theorems of the Logical Calculus 67
10. Generalization Rule and Elimination of Constants 75
11. The Completeness of the Logical Calculus 79
12. Definability 86
13. Peano Arithmetic 94
14. Skolem–Löwenheim Theorems 104
15. Ultraproducts 111
16. Types of Elements 121
17. Supplementary Questions 136

vii
PART II  Selected Topics

18. Defining Functions in $\mathbb{N}$  147
19. Total Functions  160
20. Incompleteness of Arithmetic  169
21. Arithmetical Consistency  182
22. Independence of Goodstein's Theorem  201
23. Tarski's Theorem  223
24. Matiyasevich's Theorem  233

Guide to Further Reading  252

References  254

Index  257