# Contents

**Introduction** xv

## Part I: Looking at multivariate data

1 Motivation and fundamental concepts 3
   1.1 Types of data 5
   1.2 Towards a pictorial representation 6
   1.3 Some geometrical concepts 11
      1.3.1 Two-dimensional basics 11
      1.3.2 Lines and subspaces 14
      1.3.3 Extensions to higher dimensionality 16
      1.3.4 Rotation of axes 17
      1.3.5 Latent roots and vectors 22
   1.4 Similarity, dissimilarity, and distance 24

2 One-way graphical representation of data matrices 33
   2.1 Direct two-dimensional representations 35
      2.1.1 Extensions of simple graphs 35
      2.1.2 Pictorial representations 39
      2.1.3 Function representations 43
   2.2 Representations based on subspace projection 48
      2.2.1 Geometrical models of data 48
      2.2.2 Properties of the models 50
      2.2.3 Principal component analysis: geometrical concepts 53
      2.2.4 Principal component analysis: mathematical details 60
      2.2.5 Principal component analysis: properties and practical considerations 63
      2.2.6 Principal component analysis as method of data display: illustrative example 74
      2.2.7 Other subspace projection methods 75
   2.3 Non-numerical data and missing values 83

3 Graphical methods for association or proximity matrices 86
   3.1 Direct two-dimensional representation of units, derived from numerical dissimilarities 89
      3.1.1 The dendrogram 89
      3.1.2 The minimum spanning tree 102
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2 Subspace representation of units, derived from numerical dissimilarities</td>
<td>104</td>
</tr>
<tr>
<td>3.3 Subspace representation of units, derived from ordinal data</td>
<td>113</td>
</tr>
<tr>
<td>3.4 Methods for handling asymmetric matrices</td>
<td>120</td>
</tr>
<tr>
<td>3.5 Postscript: missing values</td>
<td>125</td>
</tr>
<tr>
<td>4 Two-way graphical representation of data matrices</td>
<td>124</td>
</tr>
<tr>
<td>4.1 The singular value decomposition of a (rectangular) matrix</td>
<td>126</td>
</tr>
<tr>
<td>4.2 Representing a matrix by two sets of vectors: the biplot</td>
<td>128</td>
</tr>
<tr>
<td>4.3 Representing a matrix by one set of points and one set of vectors: preference scaling</td>
<td>130</td>
</tr>
<tr>
<td>4.4 Representing a matrix by two sets of points: multidimensional unfolding</td>
<td>132</td>
</tr>
<tr>
<td>4.5 Representing incidence matrices: correspondence analysis</td>
<td>134</td>
</tr>
<tr>
<td>5 Analytical comparison of two or more graphical representations</td>
<td>147</td>
</tr>
<tr>
<td>5.1 Comparison of two n-point configurations: procrustes analysis</td>
<td>152</td>
</tr>
<tr>
<td>5.2 Simultaneous comparison of g n-point configurations: generalized procrustes analysis</td>
<td>155</td>
</tr>
<tr>
<td>5.3 Comparison of subspaces</td>
<td>157</td>
</tr>
<tr>
<td>Part II: Samples, populations, and models</td>
<td></td>
</tr>
<tr>
<td>6 Data inspection or data analysis?</td>
<td>179</td>
</tr>
<tr>
<td>6.1 Basic concepts</td>
<td>179</td>
</tr>
<tr>
<td>6.2 Three-way scaling: individual differences analysis</td>
<td>183</td>
</tr>
<tr>
<td>6.3 Models for multivariate populations</td>
<td>193</td>
</tr>
<tr>
<td>7 Distribution theory</td>
<td>199</td>
</tr>
<tr>
<td>7.1 Basic concepts of multivariate distributions</td>
<td>199</td>
</tr>
<tr>
<td>7.2 The multivariate normal distribution</td>
<td>204</td>
</tr>
<tr>
<td>7.3 The Wishart distribution</td>
<td>206</td>
</tr>
<tr>
<td>7.4 Elliptic distributions</td>
<td>210</td>
</tr>
<tr>
<td>7.5 Tests for multivariate normality</td>
<td>211</td>
</tr>
<tr>
<td>7.6 Transformations</td>
<td>215</td>
</tr>
<tr>
<td>7.7 Multivariate analysis</td>
<td>220</td>
</tr>
</tbody>
</table>
### Contents

#### Part III: Analysing ungrouped data

8 Estimation and hypothesis testing 223
  8.1 Basic concepts and distributional results 223
  8.2 Estimation of $\mu$ and $\Sigma$ 227
  8.3 Testing hypotheses about $\mu$ and $\Sigma$ 235
  8.4 Some comments on multivariate hypothesis tests 249

9 Reduction of dimensionality: inferential aspects of descriptive methods 252
  9.1 Andrews curves 253
  9.2 Principal components 254
  9.3 Multidimensional scaling 259
  9.4 Cluster analysis 261
  9.5 Comment 264

10 Discrete data 265
  10.1 Entirely discrete data: summary and model 265
  10.2 Entirely discrete data: analysis 271
    10.2.1 Two-dimensional tables 272
    10.2.2 Three- and higher-dimensional tables 280
  10.3 Mixed discrete and continuous data 282

#### Part IV: Analysing grouped data

11 Incorporating group structure: descriptive statistics 289
  11.1 Highlighting differences between groups: canonical variates 291
  11.2 Identifying common features across groups: within-group components 306
  11.3 Partial information: identifying group membership 309
  11.4 Miscellaneous topics 312
    11.4.1 Categorical data 312
    11.4.2 Missing values 316
    11.4.3 Outliers and robustness 318
  11.5 The need for inferential methods 321

12 Inferential aspects: the two-group case 323
  12.1 Estimation and hypothesis testing 323
    12.1.1 The multivariate normal model 323
    12.1.2 The multinomial model 329
    12.1.3 The location model 330
    12.1.4 Discrimination and classification 330
  12.2 Classification rules based on probability models 332
12.2.1 Fundamental principles
12.2.2 The multivariate normal model
12.2.3 The multinomial model
12.2.4 The location model

12.3 Other classification rules
12.3.1 Fisher's linear discriminant function
12.3.2 Logistic discrimination
12.3.3 Distance-based discrimination

12.4 Evaluating the performance of an allocation rule
12.5 A practical example

13 Inferential aspects: more than two groups
13.1 The multivariate normal model
13.2 Differences between the groups: canonical variates
13.3 Treatment structure: multivariate analysis of variance
13.4 Univariate repeated measurements
13.5 Similarities between the groups: common principal components
13.6 Discrimination and classification
13.6.1 Sample space partition
13.6.2 Distance-based allocation rule
13.6.3 Logistic discrimination
13.7 Non-normal data

Part V: Analysing association among variables

14 Measuring and interpreting association
14.1 Measuring association between two variables
14.2 Interpreting association between two variables
14.3 Graphical investigation of many associations
14.4 Correcting correlations for effects of extraneous variables
14.5 Measuring association between two sets of variables
14.6 Testing hypotheses about sets of associations
14.6.1 Test that all population correlations are zero (mutual independence of all variables)
14.6.2 Test that all population canonical correlations are zero
14.6.3 Test that some population canonical correlations are zero

15 Exploiting observed associations: manifest-variable models
15.1 Motivation
15.2 Multivariate regression
## Contents

15.2.1 Introduction 454  
15.2.2 Fitting the model 457  
15.2.3 Inference about model parameters 462  
15.2.4 Assessing the adequacy of a fitted model 470  
15.3 Multivariate analysis of variance revisited 472

16 Explaining observed associations: latent-variable models 474  
16.1 Background ideas and principles 474  
16.2 Continuous manifest variables: factor analysis 477  
16.2.1 Basic model 477  
16.2.2 Implications of the model 483  
16.2.3 A data set 485  
16.2.4 Estimation of factor loadings and specific variances 486  
16.2.5 Test of goodness-of-fit, and choice of q 493  
16.2.6 Factor rotation 496  
16.2.7 Estimation of factor scores 500  
16.2.8 Dimensionality reduction using factor analysis 502  
16.2.9 Binary data 503  
16.3 Discrete manifest variables: latent-structure analysis 503  
16.3.1 Background 503  
16.3.2 Model 505  
16.3.3 Statistical aspects of the latent-class model 508  
16.4 A recent unified approach 513

17 Conclusion: some general multivariate problems 516  
17.1 Variable selection 516  
17.2 Non-parametric assessment of error: data re-sampling schemes 520

Appendix: some basic matrix theory 529  
A1 Definitions 529  
A2 Elementary arithmetic operations 531  
A3 Determinants and inverses 535  
A4 Quadratic forms 538  
A5 Latent roots and vectors 538  
A6 Matrix square root 540  
A7 Partitioned matrices 541  
A8 Vector differentiation 543

References 544

Index 558