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The Linear Theory of Elasticity. By Morton E. Gurtin, Professor of Mathematics, Carnegie-Mellon University, Pittsburgh, Pennsylvania (USA). (With 18 Figures)

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