Fundamentals of Dynamical Systems and Bifurcation Theory

Milan Medved

Mathematical Institute of the Slovak Academy of Sciences, Bratislava, Czechoslovakia

ADAM HILGER, BRISTOL, PHILADELPHIA AND NEW YORK
# CONTENTS

INTRODUCTION .............................................. vii

1 SELECTED READING ON THE FOUNDATIONS OF ALGEBRA,
TOPOLOGY, MATHEMATICAL ANALYSIS AND THE THEORY
OF DIFFERENTIAL EQUATIONS ......................... 1
1.1 Some basic concepts and notation .................. 1
1.2 Relations on a set .................................. 2
1.3 Linear, Euclidean and linear normed spaces ....... 3
1.4 Metric spaces ...................................... 5
1.5 Topological spaces .................................. 5
1.6 Matrices .......................................... 9
1.7 Linear mappings ................................... 10
1.8 Mathematical analysis ............................. 12
1.9 Differential equations ............................. 18

2 FOUNDATIONS OF THE THEORY OF DIFFERENTIABLE
MANIFOLDS AND DIFFERENTIABLE MAPPINGS .......... 22
2.1 $C^r$-manifolds .................................. 22
2.2 $C^r$-mappings .................................... 26
2.3 Tangent space to a $C^r$-manifold .................. 27
2.4 $C^r$-submanifolds ................................ 32
2.5 $C^r$-manifolds in $R^N$ ........................... 35
2.6 Immersion and submersion theorems ................. 36
2.7 Regular and critical values of mappings .......... 39
2.8 Topology on the space of $C^r$-mappings ......... 41
2.9 Jets ............................................. 45
2.10 Transversality ................................... 46
2.11 Stratification of algebraic and semi-algebraic manifolds .... 55
2.12 Transversality to stratification ................. 61

3 VECTOR FIELDS AND DYNAMICAL SYSTEMS ............ 63
3.1 Vector fields on differentiable manifolds ......... 63
3.2 Limit properties of dynamical systems .................................. 75
3.3 Examples of vector fields .............................................. 84
3.4 Generic properties of parameter-dependent matrices .................. 86
3.5 Linear dynamical systems and some notions from the theory of non-linear dynamical systems .......................... 103
3.6 Grobman–Hartman Theorem ............................................ 128
3.7 Normal forms of differential equations ............................... 143
3.8 Poincaré mapping ..................................................... 159

4 INVARIANT MANIFOLDS ..................................................... 172
4.1 Stable and unstable manifolds ......................................... 172
4.2 Centre manifolds .................................................... 183

5 GENERIC BIFURCATIONS OF VECTOR FIELDS AND DIFFEOMORPHISMS .................................................. 203
5.1 Ljapunov-Schmidt Method ............................................. 203
5.2 Generic bifurcations of 1-parameter systems of vector fields in neighbourhoods of singular points ....................... 214
5.3 Generic bifurcations of 1-parameter systems of diffeomorphisms 238
5.4 Generic bifurcations of 1-parameter systems of vector fields in neighbourhoods of periodic trajectories ............... 258

6 COMPLEMENTARY NOTES ON THE CONTEMPORARY THEORY OF DYNAMICAL SYSTEMS ......................................... 262
6.1 Generic bifurcations of multi-parameter systems of vector fields .................................................. 262
6.2 Global theory of dynamical systems .................................. 270
6.3 ŠiŠnikov bifurcation .................................................. 273
6.4 Global Hopf bifurcation ............................................. 276
6.5 Attractors and chaotic sets .......................................... 279

REFERENCES .............................................................. 282
SUBJECT INDEX ............................................................ 289