## Contents

### Chapter 1  Elements of Group Theory - 1

1.1  Set-theoretic preliminaries -1  
1.2  Groups - 2  
1.3  Algebraic operations in a group - 13  
1.4  Some subgroups of a given group G - 13  
1.5  Cosets - 15  
1.6  The class of conjugates of a complex K-17  
1.7  The Direct product of two groups - 22  
1.8  Homomorphism and Isomorphism - 23  

### Chapter 2  Some related algebraic structures - 28

2.1  Ring - 28  
2.2  Division ring - 30  
2.3  Field - 31  
2.4  Linear vector space - 31  
2.5  Linear Associative Algebra : Hyper complex system - 32  
2.6  Lie-ring and Lie-algebra - 34  

### Chapter 3  Linear Vector Space - 35

3.1  Definition - 35  
3.2  Linear dependence and independence of vectors - 36  
3.3  Change of Basis - 44  
3.4  Subspace - 46  
3.5  Isomorphism of vector spaces - 51  
3.6  On the matrix product rule - 52  
3.7  The rank of a matrix - 54  
3.8  Linear transformations - 62  
3.9  Sum and product of operators - 68  
3.10  Effect of change of basis - 69  
3.11  Active and Passive points of view 71  
3.12  The range and Kernel of a Linear transformation - 72  
3.13  Linear transformation of $R^n$ to $S^m$ - 74  
3.14  Invariant subspace-Eigenvalues and eigenvectors - 75  
3.15  Cayley-Hamilton Theorem - 78  
3.16  Euclidean space - 85  
3.17  The Schur canonical form - 92  
3.18  The Direct product of two vector spaces-The Kronecker product space - 97  
3.19  The matrix exponential - 104  
3.20  Some properties of Hermitian and Unitary matrices - 111  
3.21  The Dirac bra-ket notation - 119
Chapter 4 Elements of representation theory - 124
4.1 Definition of a representation - 124
4.2 Schur lemma - 131
4.3 Representations of the Dirac algebras $C_2$ and $C_4$ - 135
4.4 Elements of representations of Linear groups - 141
4.5 Generalised Schur lemma - 150

Chapter 5 Representations of finite groups - 152
5.1 Unitarity of a representation - 152
5.2 Orthogonality relations - 156
   a) Orthogonality relations for characters - 160
   b) Frobenius criterion for irreducibility - 161
   c) The regular representation - 162
5.3 Irreps of some finite groups - 174

Chapter 6 Representations of Linear Associative Algebras - 183
6.1 Simple and Semi-simple algebras - 183
6.2 Operator Homomorphism - 186
6.3 The fundamental theorem of Semi-simple Algebras - 187
6.4 Decomposition of $\Omega$ into two-sided Ideals - 195
   Wedderburn’s Theorem - 203
6.5 Ideal resolution and Irreps of the
   Dirac and Kemmer Algebras - 209
   The Direct product of Algebras - 234

Chapter 7 Representations of the Symmetric Group - 239
7.1 The characteristic of a permutation - 239
7.2 The number of elements in a class - 241
7.3 The Young Tableaux - 242
7.4 Lemmas for the Tableaux - 248
7.5 Young’s theorem - 252
7.6 The irreducible representations : The standard tableaux - 257
7.7 Reciprocity between the Irreps of $GL(n,C)$ and $S_f$ - 265

Chapter 8 Rotation group and its Representations - 286
8.1 Rotation matrix in terms of its axis and angle - 286
8.2 The angle and axis of an arbitrary proper orthogonal matrix - 291
8.3 The eigenvalues of a rotation matrix - 292
8.4 The canonical form of a rotation matrix - 293
8.5 The Euler resolution of a rotation - 297
   The Euler-Brauer resolution - 303
8.6 Quaternions and rotations - 305
8.7 Stereographic projection and the $SU(2)$ representation - 313
8.8 Invariant integration - 315
8.9 Irreps of the algebra $so(3)$ - 318
8.10 Exponentiation of the infinitesimal operators - 329
8.11 The character formula - 337
8.12 The $D^j$ representation through SU(2) - 338
8.13 Orthogonality and completeness of the D-functions - 344
8.14 Additional properties of the $D^j$ irreps - 346
8.15 Representation in Function space : Irreducible Teusors - 353
8.16 Differential operators for the infinitesimal transformations - spherical functions - 358
8.17 Kronecker product representation : Clebsch - Gordan theorem - 363
8.18 Clebsch - Gordan Co-efficients - 369
8.19 The Wigner - Eckart theorem - 375
Appendix
Wigner’s derivation of the C-G coefficients - 380

Chapter 9  The Crystallographic point groups - 387

9.1 Preliminaries - 387
9.2 Finite-dimensional subgroups of SO(3) - 389
9.3 The crystallographic point groups (first kind) - 404
9.4 The crystallographic point groups of the second kind - 406
9.5 The character Tables of the point groups - 417

Chapter 10 The Lorentz Group and its Representations - 425

10.1 The Lorentz transformation - 425
10.2 Minkowski space - 431
orthogonality properties of four-vectors - 432
Planes in M passing through the origin and their classification - 435
10.3 The Lorentz Group - 438
The four pieces of $L_3^3$ - 440
Subgroups of $L_3^3$ - 442
10.4 Eigenvalues and eigenvectors of an OPLT - 446
10.5 Planar transformations - 454
10.6 Canonical forms of planar OPLT’s - 468
10.7 The canonical form of an arbitrary non-null OPLT - 473
10.8 Synge’s physical interpretation of null and non-null OPLT’s - 480
10.9 OPLT as a polynomial in the ILT - 481
10.10 Determination of the blades of a screw-like OPLT - 484
10.11 Planar resolutions of an OPLT - 491
i) OPLT as a commuting product of two orthogonal planar OPLT’s - 491
ii) OPLT as a product of a rotation and a boost - 494
iii) OPLT as a product of two involutions - 499
iv) The Euler resolution of an OPLT - 500
v) The Euler - Brauer resolution of an OPLT - 504
10.12 Complex Lie-Cartan parameters of SO(3,1) - 508
10.13 Quaternions and OPLT’s - 516
The product of two boosts; Wigner Rotation - 524
The Thomas precession - 525
Chapter 10

10.14 The SL(2,C) representation of SO(3,1) - 530
10.15 Spinors - 535
10.16 The SO(3,C) representation of SO(3,1) - 543
10.17 The finite dimensional irreps of SO(3,1) - 552
Exponentiation of the infinitesimal operators - 567
The $D_{ij}^j$ irreps through SL(2,C) - 571
Special properties of the $D_{ij}^j$ irreps - 573
The Clebsch-Gordan theorem - 575
The Character formula - 576

10.18 Irreps of SO(3,1) in general - The Gelfand-Naimark basis - 578
Special properties of the $D(j_0, C)$ irreps - 586
Reality classification of the $D(j_0, C)$ irreps - 598

Chapter 11

Introduction to the Classification of
Lie-Groups-Dynkin Diagrams - 604

11.1 Preliminaries - 604
11.2 Complex extension of a real Lie-algebra - 605
11.3 Simple and Semi-simple Lie-algebras - 605
11.4 Cartan’s criterion for a Lie-algebra to be semi simple - 606
11.5 The adjoint representation - 609
11.6 Classification of Lie-Groups-Dynkin diagrams - 610
Dynkin diagrams - 613

Index 620-623