Contents

Preface v
Acknowledgments ix
Contributors xviii

1. OVERVIEW 1
Ralph B. D'Agostino and Michael A. Stephens

1.1 Goodness-of-Fit Techniques 1
1.2 Objectives of the Book 3
1.3 The Topics of the Book 4

2. GRAPHICAL ANALYSIS 7
Ralph B. D'Agostino

2.1 Introduction 7
2.2 Empirical Cumulative Distribution Function 8
2.3 General Concepts of Probability Plotting 24
2.4 Normal Probability Plotting 35
2.5 Lognormal Probability Plotting 47
2.6 Weibull Probability Plotting 54
2.7 Other Topics 57
2.8 Concluding Comment 59
References 59

References 59
CONTENTS

3. TESTS OF CHI-SQUARED TYPE 63
David S. Moore

3.1 Introduction 63
3.2 Classical Chi-Squared Statistics 64
3.3 General Chi-Squared Statistics 75
3.4 Recommendations on Use of Chi-Squared Tests 91
References 93

4. TESTS BASED ON EDF STATISTICS 97
Michael A. Stephens

4.1 Introduction 97
4.2 Empirical Distribution Function Statistics 97
4.3 Goodness-of-Fit Tests Based on the EDF (EDF Tests) 102
4.4 EDF Tests for a Fully Specified Distribution (Case 0) 104
4.5 Comments on EDF Tests for Case 0 106
4.6 Power of EDF Statistics for Case 0 110
4.7 EDF Tests for Censored Data: Case 0 111
4.8 EDF Tests for the Normal Distribution with Unknown Parameters 122
4.9 EDF Tests for the Exponential Distribution 133
4.10 EDF Tests for the Extreme-Value Distribution 145
4.11 EDF Tests for the Weibull Distribution 149
4.12 EDF Tests for the Gamma Distribution 151
4.13 EDF Tests for the Logistic Distribution 156
4.14 EDF Tests for the Cauchy Distribution 160
4.15 EDF Tests for the von Mises Distribution 164
4.16 EDF Tests for Continuous Distributions: Miscellaneous Topics 166
4.17 EDF Tests for Discrete Distributions 171
4.18 Combinations of Tests 176
4.19 EDF Statistics as Indicators of Parent Populations 180
4.20 Tests Based on Normalized Spacings 180
References 185

5. TESTS BASED ON REGRESSION AND CORRELATION 195
Michael A. Stephens

5.1 Introduction 195
5.2 Regression Tests: Models 196
5.3 Measure of Fit 197
5.4 Tests Based on the Correlation Coefficient 198
5.5 The Correlation Tests for the Uniform Distribution with Unknown Limits 199
### 5.6 The Correlation Test for U(0,1) 201
### 5.7 Regression Tests for the Normal Distribution 1 201
### 5.8 Regression Tests Based on Residuals 205
### 5.9 Tests Based on the Ratio of Two Estimates of Scale 206
### 5.10 Regression Tests for the Normal Distribution 2 207
### 5.11 Regression Tests for the Exponential Distribution 215
### 5.12 Tests Based on the Ratio of Two Estimates of Scale: Further Comments 223
### 5.13 Regression Tests for Other Distributions: General Comments 224
### 5.14 Correlation Tests for the Extreme-Value Distribution 225
### 5.15 Correlation Tests for Other Distributions 225

### References 230

### 6. SOME TRANSFORMATION METHODS IN GOODNESS-OF-FIT 235
Charles P. Quesenberry

6.1 Introduction 235
6.2 Probability Integral Transformations 239
6.3 Some Properties of CPIT's 244
6.4 Testing Simple Uniformity 246
6.5 Transformations for Particular Families 252
6.6 Numerical Examples 260

### References 275

### 7. MOMENT (\(\sqrt{b_1}, b_2\)) TECHNIQUES 279
K. O. Bowman and L. R. Shenton

7.1 Introduction 279
7.2 Normal Distribution 280
7.3 Nonnormal Sampling 287
7.4 Moments of Sample Moments 288
7.5 The Correlation Between \(\sqrt{b_1}\) and \(b_2\) 292
7.6 Simultaneous Behavior of \(\sqrt{b_1}\) and \(b_2\) 295
7.7 A Bivariate Model 306
7.8 Experimental Samples 316

### References 318

### 8. TESTS FOR THE UNIFORM DISTRIBUTION 331
Michael A. Stephens

8.1 Introduction 331
8.2 Notation 332
8.3 Transformations to Uniforms 332
8.4 Transformation from Uniforms to Uniforms 333
8.5 Superuniform Observations 334
8.6 Tests Based on the Empirical Distribution Function (EDF) 334
8.7 Regression and Correlation Tests 336
8.8 Other Tests Based on Order Statistics 336
8.9 Statistics Based on Spacings 338
8.10 Statistics for Special Alternatives 345
8.11 The Neyman-Barton Smooth Tests 351
8.12 Components of Test Statistics 355
8.13 The Effect on Test Statistics of Certain Patterns of U-Values 356
8.14 Power of Test Statistics 357
8.15 Statistics for Combining Independent Tests for Several Samples 357
8.16 Tests for a Uniform Distribution with Unknown Limits 360
8.17 Tests for Censored Uniform Samples 361
References 361

9. TESTS FOR THE NORMAL DISTRIBUTION 367
Ralph B. D'Agostino
9.1 Introduction 367
9.2 Complete Random Samples 368
9.3 Classification of Existing Tests 370
9.4 Comparisons of Tests 403
9.5 Recommendations 405
9.6 Tests of Normality on Residuals 406
9.7 Multivariate Normality 409
References 413

10. TESTS FOR THE EXPONENTIAL DISTRIBUTION 421
Michael A. Stephens
10.1 Introduction and Contents 421
10.2 Notation 424
10.3 Tests for Exponentiality: The Four Cases 425
10.4 Applications of the Exponential Distribution 426
10.5 Transformations from Exponentials to Exponentials or to Uniforms 429
10.6 Test Situations and Choice of Procedures 432
10.7 Tests with Origin Known: Groups 1, 2, and 3 435
10.8 Group 1 Tests 435
10.9 Group 2 Tests, Applied to U = JX 438
10.10 The Effect of Zero Values, and of Ties 444
10.11 Group 3 Tests Applied to $X' = NX$, or to $U' = KX$ 445
10.12 Discussion of the Data Set 451
10.13 Evaluation of Tests for Exponentiality 451
10.14 Tests with Origin and Scale Unknown 455
10.15 Summary 456
References 457

11. ANALYSIS OF DATA FROM CENSORED SAMPLES 461
John R. Michael and William R. Schucany
11.1 Introduction 461
11.2 Probability Plots 463
11.3 Testing a Simple Null Hypothesis 480
11.4 Testing a Composite Hypothesis 487
References 493

12. THE ANALYSIS AND DETECTION OF OUTLIERS 497
Gary L. Tietjen
12.1 Introduction 497
12.2 A Single Outlier in a Univariate Sample 500
12.3 Multiple Outliers in a Univariate Sample 504
12.4 The Identification of a Single Outlier in Linear Models 507
12.5 Multiple Outliers in the Linear Model 516
12.6 Accommodation of Outliers 517
12.7 Multivariate Outliers 520
12.8 Outliers in Time Series 520
References 521

APPENDIX 523
1. Table 1, Cumulative Distribution Function of the Standard Normal Distribution 524
2. Table 2, Critical Values of the Chi-Square Distribution 526
3. Simulated Data Sets 527
4. Real Data Sets 546

INDEX 551