

Armin Bunde Shlomo Havlin (Eds.)

Fractals and Disordered Systems

With 163 Figures and 10 Tables

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