Contents

1. Prime Numbers and Unique Factorization 1
   1. Introduction, 1
   2. Prime Numbers, 2
   3. Unique Factorization, 6
   4. $a_1X_1 + a_2X_2 + \cdots + a_rX_r = n$, 11
   5. The Distribution of the Primes, 17

2. Sums of Two Squares 24
   1. Introduction, 24
   2. Integers mod $m$, 25
   3. Applications of Lemma 1.4, 32
   4. Gaussian Integers, 34
   5. Farey Sequences, 39
   6. Minkowski’s Theorem, 47
   7. Method of Descent, 50
   8. Reduction of Positive Definite Binary Quadratic Forms, 55

3. Quadratic Reciprocity 63
   1. Introduction, 63
   2. Composite Moduli, 65
   3. The Legendre Symbol, 70
   4. The First Proof, 77
   5. The Gauss Lemma, 80
## CONTENTS

6. Gauss Sums, 83
7. The Ring \( \mathbb{Z}[e^{2\pi i/n}] \), 89
8. The Jacobi Symbol, 92
9. The Kronecker Symbol, 95
10. Binary Quadratic Forms, 99

### 4. Indefinite Forms

1. Introduction, 104
2. The Square Root of 2, 105
3. The Pell Equation, 108
4. \( aX^2 + bXY + cY^2 = m \), 118
5. Automorphisms, 124
6. Reduction of Indefinite Forms, 128
7. Continued Fractions, 135
8. Reduction (II), 139
9. Automorphisms (II), 142

### 5. The Class Group and Genera

1. Introduction, 147
2. The Class Group, 148
3. The Genus Group, 156
4. What Gauss Did, 161
5. Counting Ambiguous Classes, 164
6. The Ternary Form \( Y^2 - XZ \), 168
7. The Duplication Theorem, 172
8. Sums of Three Squares, 175

**Appendix A.** \( \Delta = b^2 - 4ac \) by Jean-Pierre Serre

**Appendix B.** Tables

Bibliography

Subject Index

Notation Index